GROUPS OF ORDERS 32

SUMMARY

order	# abelian	# other decomposable	# other indecomposable	TOTAL
32	7	11	33	**51**

32 = $\langle A^{32} \rangle$

32.02 = 16 × 2 = $\langle A^{16}, B^2 \rangle$

32.03 = 8 × 4 = $\langle A^8, B^4 \rangle$

32.04 = 8 × 2^2 = $\langle A^8, B^2, C^2 \rangle$

32.05 = 4^2 × 2 = $\langle A^4, B^4, C^2 \rangle$

32.06 = 4 × 2^3 = $\langle A^4, B^2, C^2, D^2 \rangle$

32.07 = 2^5 = $\langle A^2, B^2, C^2, D^2, E^2 \rangle$

32.08 = D_8 × 2^2 = $\langle A^4, B^2, C^2, D^2, BA = A^{-1}B \rangle$

32.09 = Q_8 × 2^2 = $\langle A^4, B^2 = A^2, C^2, D^2, BA = A^{-1}B \rangle$

32.10 = 16.08 × 2 = $\langle A^4, B^2, C^2, D^2, CB = A^2BC \rangle$

32.11 = 16.09 × 2 = $\langle A^4, B^2, C^2, D^2, CA = ABC \rangle$

32.12 = $D_{4,4}$ × 2 = $\langle A^4, B^4, C^2, BA = A^{-1}B \rangle$

32.13 = $M_{8,2}^{(5)}$ × 2 = $\langle A^8, B^2, C^2, BA = A^5B \rangle$

32.14 = D_8 × 4 = $\langle A^4, B^2, C^4, BA = A^{-1}B \rangle$

32.15 = Q_8 × 4 = $\langle A^4, B^2 = A^2, C^4, BA = A^{-1}B \rangle$

32.16 = ⟨A^4, B^2, C^4, $CB = A^2BC$⟩

CC	1	2	3	4	5	6	7	8	9	10	11	12
elts	1	A^2	A	A^3	C^2	A^2C^2	AC^2	A^3C^2	B A^2B	AB A^3B	BC^2 A^2BC^2	ABC^2 A^3BC^2

CC	13	14	15	16	17	18	19	20
elts	C A^2C	AC A^3C	C^3 A^2C^3	AC^3 A^3C^3	BC A^2BC	ABC A^3BC	BC^3 A^2BC^3	ABC^3 A^3BC^3	

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
#	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	3
χ_5	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	i	i	–i	–i	i	i	–i	–i	4
χ_6	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	–i	–i	i	i	–i	–i	i	i	4
χ_7	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	i	i	–i	–i	–i	–i	i	i	5
χ_8	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	–i	–i	i	i	i	i	–i	–i	5
χ_9	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	6
χ_{10}	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	1	–1	1	–1	–1	1	–1	1	7
χ_{11}	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	–1	1	–1	1	–1	1	–1	1	8
χ_{12}	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	–1	1	–1	1	1	–1	1	–1	9
χ_{13}	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	i	–i	–i	i	i	–i	–i	i	10
χ_{14}	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	–i	i	i	–i	–i	i	i	–i	10
χ_{15}	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	i	–i	–i	i	–i	i	i	–i	11
χ_{16}	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	–i	i	i	–i	i	–i	–i	i	11
χ_{17}	2	–2	2i	–2i	2	–2	2i	–2i	0	0	0	0	0	0	0	0	0	0	0	0	12
χ_{18}	2	–2	–2i	2i	2	–2	–2i	2i	0	0	0	0	0	0	0	0	0	0	0	0	12
χ_{19}	2	–2	2i	–2i	–2	2	–2i	2i	0	0	0	0	0	0	0	0	0	0	0	0	13
χ_{20}	2	–2	–2i	2i	–2	2	2i	–2	0	0	0	0	0	0	0	0	0	0	0	0	13
^	1	2	4	4	2	2	4	4	2	4	2	4	4	4	4	4	4	4	4	4	

Normal subgroups

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 13 + 14 + 15 + 16	$\mathbf{4^2}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12	$\mathbf{4 \times 2^2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18 + 19 + 20	$\mathbf{4^2}$	**2**	3	
4	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{4 \times 2}$	**4**	4	
5	1 + 2 + 3 + 4 + 11 + 12	$\mathbf{4 \times 2}$	**4**	5	Z = 𝔄
6	1 + 2 + 5 + 6 + 9 + 11 + 13 + 15 + 17 + 19	**16.09**	**2**	6	
7	1 + 2 + 5 + 6 + 10 + 12 + 13 + 15 + 18 + 20	$\mathbf{D}_{4,4}$	**2**	7	
8	1 + 2 + 5 + 6 + 9 + 11 + 14 + 16 + 18 + 20	**16.09**	**2**	8	
9	1 + 2 + 5 + 6 + 10 + 12 + 14 + 16 + 17 + 19	$\mathbf{D}_{4,4}$	**2**	9	
10	1 + 2 + 7 + 8 + 9 + 12	$\mathbf{4 \times 2}$	**4**	10	
11	1 + 2 + 7 + 8 + 10 + 11	$\mathbf{4 \times 2}$	**4**	11	
12	1 + 5	**2**	**16.08**	12	

	Classes	H	G/H	∩	
13	1 + 6	**2**	$\mathbf{4 \times 2}$	13	
14	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	$\mathbf{4 \times 2}$	**4**	1∩2	
15	1 + 2 + 3 + 4	**4**	$\mathbf{4 \times 2}$	1∩4	
16	1 + 2 + 5 + 6 + 13 + 15	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩6	
17	1 + 2 + 5 + 6 + 14 + 16	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩8	
18	1 + 2 + 7 + 8	**4**	$\mathbf{4 \times 2}$	1∩10	
19	1 + 2 + 5 + 6 + 9 + 11	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩6	
20	1 + 2 + 5 + 6 + 10 + 12	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩7	
21	1 + 2 + 5 + 6 + 17 + 19	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩6	
22	1 + 2 + 5 + 6 + 18 + 20	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩7	
23	1 + 2 + 9	$\mathbf{2^2}$	$\mathbf{4 \times 2}$	4∩6	
24	1 + 2 + 10	**4**	$\mathbf{4 \times 2}$	4∩7	
25	1 + 2 + 11	$\mathbf{2^2}$	$\mathbf{4 \times 2}$	5∩6	
26	1 + 2 + 12	**4**	$\mathbf{4 \times 2}$	5∩7	
27	1 + 2 + 5 + 6	$\mathbf{2^2}$	$\mathbf{2^3}$	16∩18	Φ
28	1 + 2	2	4×2^2	23∩24	G′

Maximal subgroups: $\mathbf{4 \times 2^2}$, $\mathbf{4^2} \times 2$, $\mathbf{16.09} \times 2$, $\mathbf{D}_{4,4} \times 2$

32.17 = ⟨A^8, B^2, C^2, $CB = A^4BC$⟩

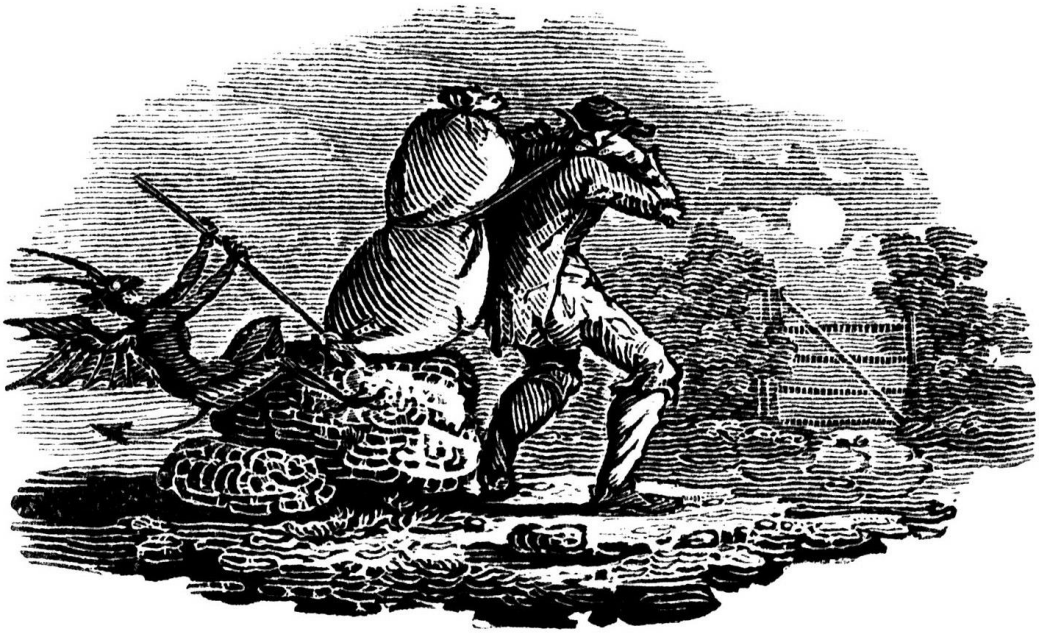
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	A^4	A^2	A^6	A	A^7	A^3	A^5	B A^4B	A^2B A^6B	AB A^5B	A^3B A^7B

13	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C A^4C	A^2C A^6C	AC A^5C	A^3C A^7C	BC A^4BC	A^2BC A^6BC	ABC A^5BC	A^3BC A^7BC

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
#	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	3
χ_5	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	4
χ_6	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	5
χ_7	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	7
χ_9	1	1	–1	–1	i	i	–i	–i	1	–1	i	–i	8
χ_{10}	1	1	–1	–1	i	i	–i	–i	1	–1	i	–i	9
χ_{11}	1	1	–1	–1	i	i	–i	–i	–1	1	–i	i	10
χ_{12}	1	1	–1	–1	i	i	–i	–i	–1	1	–i	i	11
χ_{13}	1	1	–1	–1	–i	–i	i	i	1	–1	–i	i	8
χ_{14}	1	1	–1	–1	–i	–i	i	i	1	–1	–i	i	9
χ_{15}	1	1	–1	–1	–i	–i	i	i	–1	1	i	–i	10
χ_{16}	1	1	–1	–1	–i	–i	i	i	–1	1	i	–i	11
χ_{17}	2	–2	2i	–2i	2θ	$2\theta^7$	$2\theta^3$	$2\theta^5$	0	0	0	0	0
χ_{18}	2	–2	–2i	2i	$2\theta^7$	2θ	$2\theta^5$	$2\theta^3$	0	0	0	0	0
χ_{19}	2	–2	2i	–2i	$2\theta^3$	$2\theta^5$	2θ	$2\theta^7$	0	0	0	0	0
χ_{20}	2	–2	–2i	2i	$2\theta^5$	$2\theta^3$	$2\theta^7$	2θ	0	0	0	0	0
^	**1**	**2**	**4**	**4**	**8**	**8**	**8**	**8**	**2**	**4**	**8**	**8**	

$\theta = e^{\pi i/4} = \frac{1+i}{\sqrt{2}}$.

C	13	14	15	16	17	18	19	20	
#	2	2	2	2	2	2	2	2	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	G
χ_2	–1	–1	–1	–1	–1	–1	–1	–1	1
χ_3	1	1	1	1	–1	–1	–1	–1	2
χ_4	–1	–1	–1	–1	1	1	1	1	3
χ_5	1	1	–1	–1	1	1	–1	–1	4
χ_6	–1	–1	1	1	–1	–1	1	1	5
χ_7	1	1	–1	–1	–1	–1	1	1	6
χ_8	–1	–1	1	1	1	1	–1	–1	7
χ_9	1	–1	i	–i	1	–1	i	–i	8
χ_{10}	–1	1	–i	i	–1	1	–i	i	9
χ_{11}	1	–1	i	–i	–1	1	–i	i	10
χ_{12}	–1	1	–i	i	1	–1	i	–i	11
χ_{13}	1	–1	–i	i	1	–1	–i	i	8
χ_{14}	–1	1	i	–i	–1	1	i	–i	9
χ_{15}	1	–1	–i	i	–1	1	i	–i	10
χ_{16}	–1	1	i	–i	1	–1	–i	i	11
χ_{17}	0	0	0	0	0	0	0	0	0
χ_{18}	0	0	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	0	0
χ_{20}	0	0	0	0	0	0	0	0	0
^	2	4	8	8	4	2	8	8	



Normal subgroups

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12	**8 × 2**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 13 + 14 + 15 + 16	**8 × 2**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18 + 19 + 20	**8 × 2**	**2**	3	
4	1 + 2 + 3 + 4 + 9 + 10 + 13 + 14 + 17 + 18	**16.08**	**2**	4	
5	1 + 2 + 3 + 4 + 9 + 10 + 15 + 16 + 19 + 20	$\mathbf{M}_{8,2}^{(5)}$	**2**	5	
6	1 + 2 + 3 + 4 + 11 + 12 + 13 + 14 + 19 + 20	$\mathbf{M}_{8,2}^{(5)}$	**2**	6	
7	1 + 2 + 3 + 4 + 11 + 12 + 15 + 16 + 17 + 18	$\mathbf{M}_{8,2}^{(5)}$	**2**	7	
8	1 + 2 + 9 + 13 + 17	$\mathbf{D}_8$	**4**	8	
9	1 + 2 + 9 + 14 + 18	$\mathbf{D}_8$	**4**	9	
10	1 + 2 + 10 + 13 + 18	$\mathbf{D}_8$	**4**	10	
11	1 + 2 + 10 + 14 + 17	$\mathbf{Q}_8$	**4**	11	
12	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**8**	$\mathbf{2^2}$	1∩2	**Z** = $\mathfrak{Z}$
13	1 + 2 + 3 + 4 + 9 + 10	**4 × 2**	$\mathbf{2^2}$	1∩4	
14	1 + 2 + 3 + 4 + 11 + 12	**8**	$\mathbf{2^2}$	1∩6	
15	1 + 2 + 9	$\mathbf{2^2}$	**4 × 2**	1∩8	

	Classes	H	G/H		
16	1 + 2 + 10	**4**	**4 × 2**	1∩10	
17	1 + 2 + 3 + 4 + 13 + 14	**4 × 2**	$\mathbf{2^2}$	2∩4	
18	1 + 2 + 3 + 4 + 15 + 16	**8**	$\mathbf{2^2}$	2∩5	
19	1 + 2 + 13	$\mathbf{2^2}$	**4 × 2**	2∩8	
20	1 + 2 + 14	**4**	**4 × 2**	2∩9	
21	1 + 2 + 3 + 4 + 17 + 18	**4 × 2**	$\mathbf{2^2}$	3∩4	
22	1 + 2 + 3 + 4 + 19 + 20	**8**	$\mathbf{2^2}$	3∩5	
23	1 + 2 + 17	**4**	**4 × 2**	3∩8	
24	1 + 2 + 18	$\mathbf{2^2}$	**4 × 2**	3∩9	
25	1 + 2 + 3 + 4	**4**	$\mathbf{2^3}$	13∩14	
26	1 + 2	**2**	$\mathbf{4 \times 2^2}$	15∩16	**G′**

Maximal subgroups: [**8 × 2**] × 3, **16.08**, [$\mathbf{M}_{8,2}^{(5)}$] × 3

32.18 = ⟨A^2, B^4, C^4, $CB = ABC$⟩

1	2	3	4	5	6	7	8	9	10	11	12
1	A	B^2	AB^2	C^2	AC^2	B^2C^2	AB^2C^2	B AB	B^3 AB^3	BC^2 ABC^2	B^3C^2 AB^3C^2

13	14	15	16	17	18#s	19	20
C AC	B^2C AB^2C	C^3 AC^3	B^2C^3 AB^2C^3	BC ABC	B^3C AB^3C	BC^3 ABC^3	B^3C^3 AB^3C^3

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
#	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	3
χ_5	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	4
χ_6	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	4
χ_7	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	5
χ_8	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	5
χ_9	1	1	–1	–1	1	1	–1	–1	–i	i	–i	i	6
χ_{10}	1	1	–1	–1	1	1	–1	–1	–i	i	–i	i	7
χ_{11}	1	1	–1	–1	1	1	–1	–1	i	–i	i	–i	6
χ_{12}	1	1	–1	–1	1	1	–1	–1	i	–i	i	–i	7
χ_{13}	1	1	–1	–1	–1	–1	1	1	–i	i	i	–i	8
χ_{14}	1	1	–1	–1	–1	–1	1	1	–i	i	i	–i	9
χ_{15}	1	1	–1	–1	–1	–1	1	1	i	–i	–i	i	9
χ_{16}	1	1	–1	–1	–1	–1	1	1	i	–i	–i	i	8
χ_{17}	2	–2	2	–2	2	–2	2	–2	0	0	0	0	10
χ_{18}	2	–2	2	–2	–2	2	–2	2	0	0	0	0	11
χ_{19}	2	–2	–2	2	2	–2	–2	2	0	0	0	0	12
χ_{20}	2	–2	–2	2	–2	2	2	–2	0	0	0	0	13
^	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	

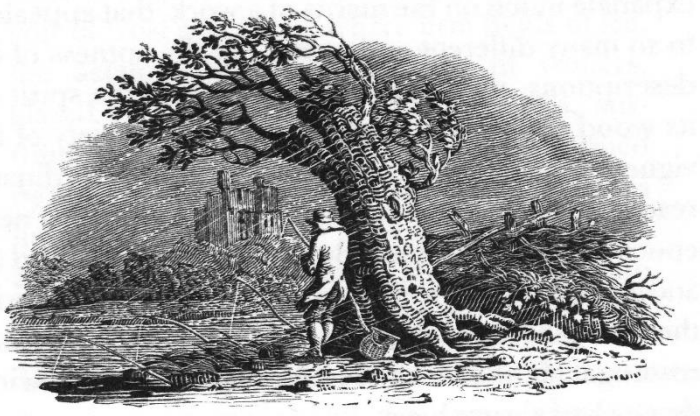
C	13	14	15	16	17	18	19	20	
#	2	2	2	2	2	2	2	2	ℵ
χ_1	1	1	1	1	1	1	1	1	G
χ_2	–1	–1	–1	–1	–1	–1	–1	–1	1
χ_3	1	1	1	1	–1	–1	–1	–1	2
χ_4	–1	–1	–1	–1	1	1	1	1	3
χ_5	–i	–i	i	i	–i	–i	i	i	4
χ_6	i	i	–i	–i	i	i	–i	–i	4
χ_7	–i	–i	i	i	i	i	–i	–i	5
χ_8	i	i	–i	–i	–i	–i	i	i	5
χ_9	1	–1	1	–1	–i	i	–i	i	6
χ_{10}	–1	1	–1	1	i	–i	i	–i	7
χ_{11}	1	–1	1	–1	i	–i	i	–i	6
χ_{12}	–1	1	–1	1	–i	i	–i	i	7
χ_{13}	–i	i	i	–i	–1	1	1	–1	8
χ_{14}	i	–i	–i	i	1	–1	–1	1	9
χ_{15}	–i	i	i	–i	1	–1	–1	1	9
χ_{16}	i	–i	–i	i	–1	1	1	–1	8
χ_{17}	0	0	0	0	0	0	0	0	10
χ_{18}	0	0	0	0	0	0	0	0	11
χ_{19}	0	0	0	0	0	0	0	0	12
χ_{20}	0	0	0	0	0	0	0	0	13
^	4	4	4	4	4	4	4	4	

Normal subgroups

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12	$\mathbf{4 \times 2^2}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 13 + 14 + 15 +16	$\mathbf{4 \times 2^2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18 + 19 + 20	$\mathbf{4 \times 2^2}$	**2**	3	
4	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{4 \times 2}$	**4**	4	
5	1 + 2 + 3 + 4 + 11 + 12	$\mathbf{4 \times 2}$	**4**	5	
6	1 + 2 + 5 + 6 + 13 + 15	$\mathbf{4 \times 2}$	**4**	6	
7	1 + 2 + 5 + 6 + 14 + 16	$\mathbf{4 \times 2}$	**4**	7	
8	1 + 2 + 7 + 8 + 18 + 19	$\mathbf{4 \times 2}$	**4**	8	
9	1 + 2 + 7 + 8 + 17 + 20	$\mathbf{4 \times 2}$	**4**	9	
10	1 + 3 + 5 + 7	$\mathbf{2^2}$	$\mathbf{D_8}$	10	
11	1 + 3 + 6 + 8	$\mathbf{2^2}$	$\mathbf{D_8}$	11	
12	1 + 4 + 5 + 8	$\mathbf{2^2}$	$\mathbf{D_8}$	12	
13	1 + 4 + 6 + 7	$\mathbf{2^2}$	$\mathbf{Q_8}$	13	
14	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	$\mathbf{2^3}$	$\mathbf{2^2}$	1∩2	$\mathbf{Z = \Phi = \mathfrak{N}}$
15	1 + 2 + 5 + 6	$\mathbf{2^2}$	$\mathbf{4 \times 2}$	1∩6	
16	1 + 2 + 7 + 8	$\mathbf{2^2}$	$\mathbf{4 \times 2}$	1∩8	
17	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{4 \times 2}$	2∩4	

	Classes	H	G/H	∩	
18	1 + 2	**2**	$\mathbf{4^2}$	4∩6	**G′**
19	1 + 3	**2**	**16.09**	4∩10	
20	1 + 4	**2**	**16.10**	5∩12	
21	1 + 5	**2**	**16.09**	6∩10	
22	1 + 6	**2**	**16.10**	6∩11	
23	1 + 7	**2**	**16.10**	8∩10	
24	1 + 8	**2**	**16.09**	8∩11	

Maximal subgroups: $[\mathbf{4 \times 2^2}] \times 3$



32.19 = $M_{8,4}^{(5)} = \langle A^8, B^4, BA = A^5B \rangle$

1	2	3	4	5	6	7	8	9	10	11	12
1	A^4	A^2	A^6	B^2	A^4B^2	A^2B^2	A^6B^2	B A^4B	A^2B A^6B	B^3 A^4B^3	A^2B^3 A^6B^3

13	14	15	16	17	18	19	20
A A^5	A^3 A^7	AB^2 A^5B^2	A^3B^2 A^7B^2	AB A^5B	A^3B A^7B	AB^3 A^5B^3	A^3B^3 A^7B^3

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	1	1	1	1	1	1	1	2	2	2	2	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	3
χ_5	1	1	1	1	–1	–1	–1	–1	i	i	–i	–i	4
χ_6	1	1	1	1	–1	–1	–1	–1	i	i	–i	–i	5
χ_7	1	1	1	1	–1	–1	–1	–1	–i	–i	i	i	4
χ_8	1	1	1	1	–1	–1	–1	–1	–i	–i	i	i	5
χ_9	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	6
χ_{10}	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	6
χ_{11}	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	7
χ_{12}	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	7
χ_{13}	1	1	–1	–1	–1	–1	1	1	i	–i	–i	i	8
χ_{14}	1	1	–1	–1	–1	–1	1	1	i	–i	–i	i	9
χ_{15}	1	1	–1	–1	–1	–1	1	1	–i	i	i	–i	9
χ_{16}	1	1	–1	–1	–1	–1	1	1	–i	i	i	–i	8
χ_{17}	2	–2	2i	–2i	2	–2	2i	–2i	0	0	0	0	10
χ_{18}	2	–2	2i	–2i	–2	2	–2i	2i	0	0	0	0	11
χ_{19}	2	–2	–2i	2i	2	–2	–2i	2i	0	0	0	0	10
χ_{20}	2	–2	–2i	2i	–2	2	2i	–2i	0	0	0	0	11
^	1	2	4	4	2	2	4	4	4	4	4	4	

C	13	14	15	16	17	18	19	20	
#	2	2	2	2	2	2	2	2	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	G
χ_2	–1	–1	–1	–1	–1	–1	–1	–1	1
χ_3	1	1	1	1	–1	–1	–1	–1	2
χ_4	–1	–1	–1	–1	1	1	1	1	3
χ_5	1	1	–1	–1	i	i	–i	–i	4
χ_6	–1	–1	1	1	–i	–i	i	i	5
χ_7	1	1	–1	–1	–i	–i	i	i	4
χ_8	–1	–1	1	1	i	i	–i	–i	5
χ_9	i	–i	i	–i	i	–i	i	–i	6
χ_{10}	–i	i	–i	i	–i	i	–i	i	6
χ_{11}	i	–i	i	–i	–i	i	–i	i	7
χ_{12}	–i	i	–i	i	i	–i	i	–i	7
χ_{13}	i	–i	–i	i	–1	1	1	–1	8
χ_{14}	–i	i	i	–i	1	–1	–1	1	9
χ_{15}	i	–i	–i	i	1	–1	–1	1	9
χ_{16}	–i	i	i	–i	–1	1	1	–1	8
χ_{17}	0	0	0	0	0	0	0	0	10
χ_{18}	0	0	0	0	0	0	0	0	11
χ_{19}	0	0	0	0	0	0	0	0	10
χ_{20}	0	0	0	0	0	0	0	0	11
^	**8**	**8**	**8**	**8**	**8**	**8**	**8**	**8**	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12	$\mathbf{4^2}$	**2**	1	$\mathfrak{N}$
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 13 + 14 + 15 + 16	**8 × 2**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18 + 19 + 20	**8 × 2**	**2**	3	
4	1 + 2 + 3 + 4 + 13 + 14	**8**	**4**	4	
5	1 + 2 + 3 + 4 + 15 + 16	**8**	**4**	5	
6	1 + 2 + 5 + 6 + 9 + 11	**8**	**4**	6	
7	1 + 2 + 5 + 6 + 10 + 12	**8**	**4**	7	
8	1 + 2 + 7 + 8 + 18 + 19	**8**	**4**	8	
9	1 + 2 + 7 + 8 + 17 +20	**8**	**4**	9	
10	1 + 5	**2**	$\mathbf{M_{8,2}^{(5)}}$	10	
11	1 + 6	**2**	$\mathbf{M_{8,2}^{(5)}}$	11	
12	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**4 × 2**	$\mathbf{2^2}$	1∩2	**Z = Φ**
13	1 + 2 +3 + 4	**4**	**4 × 2**	1∩4	
14	1 + 2 + 7 + 8	**4**	**4 × 2**	1∩8	
15	1 + 2 + 5 + 6			2∩6	
16	1 + 2	**2**	$\mathbf{4^2}$	4∩6	**G′**

Maximal subgroups: [$\mathbf{4^2}$], [**8 × 2**] × 2

32.20 = $\langle A^8, B^2, C^2, CA = ABC \rangle$

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	B	A^4	A^4B	A^2	A^2B	A^6	A^6B	C BC	A^4C A^4BC	A^2C A^2BC	A^6C A^6BC

13	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A AB	A^5 A^5B	A^3 A^3B	A^7 A^7B	AC ABC	A^5C A^5BC	A^3C A^3BC	A^7C A^7BC

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
#	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	3
χ_5	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	4
χ_6	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	4
χ_7	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	5
χ_8	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	5
χ_9	1	1	–1	–1	i	i	–i	–i	1	–1	i	–i	6
χ_{10}	1	1	–1	–1	i	i	–i	–i	1	–1	i	–i	6
χ_{11}	1	1	–1	–1	i	i	–i	–i	–1	1	–i	i	7
χ_{12}	1	1	–1	–1	i	i	–i	–i	–1	1	–i	i	7
χ_{13}	1	1	–1	–1	–i	–i	i	I	1	–1	–i	i	8
χ_{14}	1	1	–1	–1	–i	–i	i	I	1	–1	–i	i	8
χ_{15}	1	1	–1	–1	–i	–i	i	I	–1	1	i	–i	7
χ_{16}	1	1	–1	–1	–i	–i	i	I	–1	1	i	–i	7
χ_{17}	2	–2	2	–2	2	–2	2	–2	0	0	0	0	9
χ_{18}	2	–2	2	–2	–2	2	–2	2	0	0	0	0	10
χ_{19}	2	–2	–2	2	2i	–2i	–2i	2i	0	0	0	0	11
χ_{20}	2	–2	–2	2	–2i	2i	2i	–2i	0	0	0	0	11
^	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**2**	**2**	**4**	**4**	

$\theta = e^{2\pi i/8} = \dfrac{1+i}{\sqrt{2}}$.

C	13	14	15	16	17	18	19	20	
#	2	2	2	2	2	2	2	2	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	G
χ_2	–1	–1	–1	–1	–1	–1	–1	–1	1
χ_3	1	1	1	1	–1	–1	–1	–1	2
χ_4	–1	–1	–1	–1	1	1	1	1	3
χ_5	i	i	–i	–i	i	I	–i	–i	4
χ_6	–i	–i	i	i	–i	–i	i	I	4
χ_7	i	i	–i	–i	–i	–i	i	I	5
χ_8	–i	–i	i	i	i	I	–i	–i	5
χ_9	θ	θ^5	θ^3	θ^7	θ	θ^5	θ^3	θ^7	6
χ_{10}	θ^5	θ	θ^7	θ^3	θ^5	θ	θ^7	θ^3	6
χ_{11}	θ	θ^5	θ^3	θ^7	θ^5	θ	θ^7	θ^3	7
χ_{12}	θ^5	θ	θ^7	θ^3	θ	θ^5	θ^3	θ^7	7
χ_{13}	θ^7	θ^3	θ^5	θ	θ^7	θ^3	θ^5	θ	8
χ_{14}	θ^3	θ^7	θ	θ^5	θ^3	θ^7	θ	θ^5	8
χ_{15}	θ^7	θ^3	θ^5	θ	θ^3	θ^7	θ	θ^5	7
χ_{16}	θ^3	θ^7	θ	θ^5	θ^7	θ^3	θ^5	θ	7
χ_{17}	0	0	0	0	0	0	0	0	9
χ_{18}	0	0	0	0	0	0	0	0	10
χ_{19}	0	0	0	0	0	0	0	0	11
χ_{20}	0	0	0	0	0	0	0	0	11
^	**8**	**8**	**8**	**8**	**8**	**8**	**8**	**8**	

	Classes	H	G/H	$\cap$	
1	1+2+3+4+5+6+7+8+9+10+11+12	$\mathbf{4 \times 2^2}$	**2**	1	
2	1+2+3+4+5+6+7+8+13+14+15+16	$\mathbf{8 \times 2}$	**2**	2	
3	1+2+3+4+5+6+7+8+17+18+19+20	$\mathbf{8 \times 2}$	**2**	3	
4	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{2^3}$	**4**	4	
5	1 + 2 + 3 + 4 + 11 + 12	$\mathbf{4 \times 2}$	**4**	5	
6	1 + 2	**2**	$\mathbf{8 \times 2}$	6	
7	1 + 2 + 10	$\mathbf{2^2}$	**8**	7	
8	1 + 2 + 9	$\mathbf{2^2}$	**8**	8	
9	1 + 3 + 5 + 7	**4**	$\mathbf{2^3}$	9	
10	1 + 3 + 6 + 8	**4**	$\mathbf{D_8}$	10	
11	1 + 4	**2**	$\mathbf{M_{8,2}^{(5)}}$	11	
12	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1$\cap$2	
13	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{4 \times 2}$	2$\cap$4	$\mathbf{Z = \Phi = \mathfrak{N}}$
14	1 + 3	**2**	**16.09**	4$\cap$9	$\mathbf{G'}$

Maximal subgroups: $[\mathbf{4 \times 2^2}]$, $[\mathbf{8 \times 2}] \times 2$

32.21 = $D_{4,8} = \langle A^4, B^8, BA = A^{-1}B \rangle$

1	2	3	4	5	6	7	8	9	10	11	12
1	A^2	B^4	A^2B^4	B^2	A^2B^2	B^6	A^2B^6	B A^2B	B^7 A^2B^7	B^3 A^2B^3	B^5 A^2B^5

13	14	15	16	17	18	19	20
A A^3	AB^4 A^3B^4	AB^2 A^3B^2	AB^6 A^3B^6	AB A^3B	AB^7 A^3B^7	AB^3 A^3B^3	AB^5 A^3B^5

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
#	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	–1	–1	–1	–1	i	–i	–i	i	2
χ_4	1	1	1	1	–1	–1	–1	–1	–i	i	I	–i	2
χ_5	1	1	–1	–1	i	i	–i	–i	θ	θ^7	θ^3	θ^5	3
χ_6	1	1	–1	–1	–i	–i	i	i	θ^7	θ	θ^5	θ^3	3
χ_7	1	1	–1	–1	–i	–i	i	i	θ^3	θ^5	θ	θ^7	3
χ_8	1	1	–1	–1	i	i	–i	–i	θ^5	θ^3	θ^7	θ	3
χ_9	1	1	1	1	1	1	1	1	1	1	1	1	4
χ_{10}	1	1	1	1	1	1	1	1	–1	–1	–1	–1	5
χ_{11}	1	1	1	1	–1	–1	–1	–1	i	–i	–i	i	6
χ_{12}	1	1	1	1	–1	–1	–1	–1	–i	i	I	–i	7
χ_{13}	1	1	–1	–1	i	i	–i	–i	θ	θ^7	θ^3	θ^5	8
χ_{14}	1	1	–1	–1	–i	–i	i	i	θ^7	θ	θ^5	θ^3	9
χ_{15}	1	1	–1	–1	–i	–i	i	i	θ^3	θ^5	θ	θ^7	9
χ_{16}	1	1	–1	–1	i	i	–i	–i	θ^5	θ^3	θ^7	θ	9
χ_{17}	2	–2	2	–2	2	–2	2	–2	0	0	0	0	10
χ_{18}	2	–2	2	–2	–2	2	–2	2	0	0	0	0	11
χ_{19}	2	–2	–2	2	2i	–2i	–2i	2i	0	0	0	0	12
χ_{20}	2	–2	–2	2	–2i	2i	2i	–2i	0	0	0	0	12
^	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**8**	**8**	**8**	**8**	

$\theta = e^{2\pi i/8} = \dfrac{1+i}{\sqrt{2}}$.

C	13	14	15	16	17	18	19	20	
#	2	2	2	2	2	2	2	2	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	–1	–1	i	–i	–i	i	2
χ_4	1	1	–1	–1	–i	i	i	–i	2
χ_5	1	–1	i	–i	θ	θ^7	θ^3	θ^5	3
χ_6	1	–1	–i	i	θ^7	θ	θ^5	θ^3	3
χ_7	1	–1	–i	i	θ^3	θ^5	θ	θ^7	3
χ_8	1	–1	i	–i	θ^5	θ^3	θ^7	θ	3
χ_9	–1	–1	–1	–1	–1	–1	–1	–1	4
χ_{10}	–1	–1	–1	–1	1	1	1	1	5
χ_{11}	–1	–1	1	1	–i	i	i	–i	6
χ_{12}	–1	–1	1	1	i	–i	–i	i	7
χ_{13}	–1	1	–i	i	$-\theta$	$-\theta^7$	$-\theta^3$	$-\theta^5$	8
χ_{14}	–1	1	i	–i	$-\theta^7$	$-\theta$	$-\theta^5$	$-\theta^3$	9
χ_{15}	–1	1	i	–i	$-\theta^3$	$-\theta^5$	$-\theta$	$-\theta^7$	9
χ_{16}	–1	1	–i	i	$-\theta^5$	$-\theta^3$	$-\theta^7$	$-\theta$	9
χ_{17}	0	0	0	0	0	0	0	0	10
χ_{18}	0	0	0	0	0	0	0	0	11
χ_{19}	0	0	0	0	0	0	0	0	12
χ_{20}	0	0	0	0	0	0	0	0	12
^	4	4	4	4	8	8	8	8	

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+13+14+15+16	4^2	2	**1**	
2	1 + 2 + 3 + 4 + 13 + 14	4×2	4	**2**	
3	1 + 2 + 13	4	8	**3**	
4	1+2+3+4+5+6+7+8+9+10+11+12	8×2	2	**4**	
5	1+2+3+4+5+6+7+8+17+18+19+20	4×2	2^2	**5**	**Z = Φ = $\mathfrak{R}$**
6	1 + 2 + 3 + 4 + 17 + 18 + 19 + 20			**6**	
7	1 + 2 + 3 + 4 + 15 + 16	4×2	4	**7**	
8	1 + 2 + 15 + 16			**8**	
9	1 + 2 + 14	4	8	**9**	
10	1 + 3 + 5 + 7	4	$\mathbf{D}_8$	**10**	
11	1 + 3 + 6 + 8	4	$\mathbf{Q}_8$	**11**	
12	1 + 4	2	$\mathbf{M}_{8,2}^{(5)}$	**12**	
13	1 + 2 + 3 + 4	2^2	4×2	**1∩6**	
14	1 + 2	2	8×2	**2∩8**	**G′**
15	1 + 3	2	$\mathbf{D}_{4.4}$	**2∩10**	

Maximal subgroups: [$\mathbf{4^2}$], [$\mathbf{8 \times 2}$] $\times$ 2

32.22 = $M_{16,2}^{(9)} = \langle A^{16}, B^2, BA = A^9B \rangle$

1	2	3	4	5	6	7	8	9	10	11	12
1	A^8	A^4	A^{12}	A^2	A^{10}	A^6	A^{14}	B A^8B	A^4B $A^{12}B$	A^2B $A^{10}B$	A^6B $A^{14}B$

13	14	15	16	17	18	19	20
A A^9	A^5 A^{13}	A^3 A^{11}	A^7 A^{15}	AB A^9B	A^5B $A^{13}B$	A^3B $A^{11}B$	A^7B $A^{15}B$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
#	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	1	1	1	–1	–1	–1	–1	3
χ_5	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	4
χ_6	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	4
χ_7	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	5
χ_8	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	5
χ_9	1	1	–1	–1	–i	–i	i	i	1	–1	–i	i	6
χ_{10}	1	1	–1	–1	–i	–i	i	i	1	–1	–i	i	6
χ_{11}	1	1	–1	–1	–i	–i	i	i	–1	1	i	–i	7
χ_{12}	1	1	–1	–1	–i	–i	i	i	–1	1	i	–i	7
χ_{13}	1	1	–1	–1	i	i	–i	–i	1	–1	i	–i	6
χ_{14}	1	1	–1	–1	i	i	–i	–i	1	1	i	–i	6
χ_{15}	1	1	–1	–1	i	i	–i	–i	–1	1	–i	i	7
χ_{16}	1	1	–1	–1	i	i	–i	–i	–1	1	–i	i	7
χ_{17}	2	–2	2i	–2i	2θ	$2\theta^5$	$2\theta^3$	$2\theta^7$	0	0	0	0	0
χ_{18}	2	–2	2i	–2i	$2\theta^5$	2θ	$2\theta^7$	$2\theta^3$	0	0	0	0	0
χ_{19}	2	–2	–2i	2i	$2\theta^3$	$2\theta^7$	2θ	$2\theta^5$	0	0	0	0	0
χ_{20}	2	–2	–2i	2i	$2\theta^7$	$2\theta^3$	$2\theta^5$	2θ	0	0	0	0	0
^	**1**	**2**	**4**	**4**	**8**	**8**	**8**	**8**	**2**	**4**	**8**	**8**	

$\theta = e^{2\pi i/8} = \dfrac{1+i}{\sqrt{2}}$.

C	13	14	15	16	17	18	19	20	
#	2	2	2	2	2	2	2	2	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	G
χ_2	−1	−1	−1	−1	−1	−1	−1	−1	1
χ_3	1	1	1	1	−1	−1	−1	−1	2
χ_4	−1	−1	−1	−1	1	1	1	1	3
χ_5	−i	−i	i	i	−i	−i	i	i	4
χ_6	i	i	−i	−i	i	i	−i	−i	4
χ_7	−i	−i	i	i	i	i	−i	−i	5
χ_8	i	i	−i	−i	−i	−i	i	i	5
χ_9	θ^7	θ^3	θ^5	θ	θ^7	θ^3	θ^5	θ	6
χ_{10}	θ^3	θ^7	θ	θ^5	θ^3	θ^7	θ	θ^5	6
χ_{11}	θ^7	θ^3	θ^5	θ	θ^3	θ^7	θ	θ^5	7
χ_{12}	θ^3	θ^7	θ	θ^5	θ^7	θ^3	θ^5	θ	71
χ_{13}	θ	θ^5	θ^3	θ^7	θ	θ^5	θ^3	θ^7	6
χ_{14}	θ^5	θ	θ^7	θ^3	θ^5	θ	θ^7	θ^3	6
χ_{15}	θ	θ^5	θ^3	θ^7	θ^5	θ	θ^7	θ^3	7
χ_{16}	θ^5	θ	θ^7	θ^3	θ	θ^5	θ^3	θ^7	7
χ_{17}	0	0	0	0	0	0	0	0	0
χ_{18}	0	0	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	0	0
χ_{20}	0	0	0	0	0	0	0	0	0
^	16	16	16	16	16	16	16	16	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12	8×2	2	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 13 + 14 + 15 + 16	16	2	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17 + 18 + 19 + 20	16	2	3	
4	1 + 2 + 3 + 4 + 9 + 10	4×2	4	4	$\mathfrak{N}$
5	1 + 2 + 3 + 4 + 11 + 12	8	4	5	
6	1 + 2 + 9	2^2	8	6	
7	1 + 2 + 10	4	8	7	
8	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	8	2^2	1∩2	**Z = Φ**
9	1 + 2 + 3 + 4	4	4×2	2∩4	
10	1 + 2	2	8×2	2∩6	**G′**

Maximal subgroups: [**8 × 2**], [**16**] × 2

32.23 = $D_{16} \times 2 = \langle A^8, B^2, C^2, BA = A^{-1}B \rangle$

32.24 = $M_{8,2}^{(3)} \times 2 = \langle A^8, B^2, C^2, BA = A^3B \rangle$

32.25 = $Q_{16} \times 2 = \langle A^8, B^2 = A^4, C^2, BA = A^{-1}B \rangle$

32.26 = $\langle A^8, B^2 = A^4, C^2, CA = A^{-1}C \rangle$

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	A^4	B	A^4B	A^2 A^6	A^6B A^7B	A A^7	A^3 A^5	AB A^7B	A^3B A^5B	$A^{2n}C$	$A^{2n}BC$	$A^{2n+1}C$	$A^{2n+1}BC$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	
#	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	–1	–1	1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	–1	–1	1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	–1	–1	1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	2	2	–2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2i	–2i	0	0	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}i$	$-\sqrt{2}i$	0	0	0	0	0
χ_{12}	2	–2	2i	–2i	0	0	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}i$	$\sqrt{2}i$	0	0	0	0	0
χ_{13}	2	–2	–2i	2i	0	0	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}i$	$\sqrt{2}i$	0	0	0	0	0
χ_{14}	2	–2	–2i	2i	0	0	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}i$	$-\sqrt{2}i$	0	0	0	0	0
^	**1**	**2**	**4**	**4**	**4**	**2**	**8**	**8**	**8**	**8**	**2**	**4**	**2**	**4**	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	**8 × 2**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	**4 × 2^2**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	**4 × 2^2**	**2**	3	
4	1 + 2 + 5 + 7 + 8 + 11 + 13	**D_{16}**	**2**	4	
5	1 + 2 + 5 + 7 + 8 + 12 + 14	**Q_{16}**	**2**	5	
6	1 + 2 + 5 + 9 + 10 + 11 + 14	**$M_{8,2}^{(3)}$**	**2**	6	
7	1 + 2 + 5 + 9 + 10 + 12 + 13	**$M_{8,2}^{(3)}$**	**2**	7	
8	1 + 2 + 3 + 4	**4**	**D_8**	8	**Z = $\mathfrak{R}$**
9	1 + 2	**2**	**D_8 × 2**	9	
8	1 + 2 + 5 + 7 + 8	**8**	**2^2**	1∩4	
9	1 + 2 + 5 + 9 + 10	**8**	**2^2**	1∩6	
10	1 + 2 + 3 + 4 + 5 + 6	**4 × 2**	**2^2**	1∩2	**Z_2**
11	1 + 2 + 5 + 11	**D_8**	**2^2**	2∩4	
12	1 + 2 + 5 + 13	**D_8**	**2^2**	4∩7	
13	1 + 2 + 5 + 12	**Q_8**	**2^2**	2∩5	
14	1 + 2 + 5 + 14	**Q_8**	**2^2**	5∩6	
15	1 + 2 + 5	**4**	**2^3**	8∩9	**G′ = Φ**

Maximal subgroups: [**8 × 2**], [**16.08**] × 2, [D_{16}], [$M_{8,3}^{(3)}$] × 2, [Q_{16}]

32.27 = ⟨A^8, $B^2 = A^4$, C^2, $CA = A^{-1}C$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^4	B	A^4B	A^2 A^6	A^6B A^2B	A A^7	A^3 A^5	AB A^7B	A^3B A^5B	$A^{2n}C$	$A^{2n}BC$	$A^{2n+1}C$	$A^{2n}BC$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	–1	–1	1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	–1	–1	1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	–1	–1	1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	2	2	–2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2	–2	0	0	√2	–√2	√2i	–√2i	0	0	0	0	10
χ_{12}	2	–2	2	–2	0	0	–√2	√2	–√2i	√2i	0	0	0	0	10
χ_{13}	2	–2	–2	2	0	0	√2	–√2	–√2i	√2i	0	0	0	0	11
χ_{14}	2	–2	–2	2	0	0	–√2	√2	√2i	–√2i	0	0	0	0	11
^	1	2	4	4	4	2	8	8	8	8	2	4	2	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	**8 × 2**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	**4 × 2^2**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	**4 × 2^2**	**2**	3	
4	1 + 2 + 5 + 7 + 8 + 11 + 13	**D_{16}**	**2**	4	
5	1 + 2 + 5 + 7 + 8 + 12 + 14	**Q_{16}**	**2**	5	
6	1 + 2 + 5 + 9 + 10 + 11 + 14	**$M_{8,2}^{(3)}$**	**2**	6	
7	1 + 2 + 5 + 9 + 10 + 12 + 13	**$M_{8,2}^{(3)}$**	**2**	7	
8	1 + 2 + 3 + 4	**4**	**D_8**	8	**Z = $\mathfrak{N}$**
9	1 + 2 + 6	**2^2**	**D_8**	9	
10	1 + 3	**2**	**D_{16}**	10	
11	1 + 4	**2**	**D_{16}**	11	
8	1 + 2 + 5 + 7 + 8	**8**	**2^2**	1∩4	
9	1 + 2 + 5 + 9 + 10	**8**	**2^2**	1∩5	
10	1 + 2 + 3 + 4 + 5 + 6	**4 × 2**	**2^2**	1∩2	**Z_2 = Φ**
11	1 + 2 + 5 + 11	**D_8**	**2^2**	2∩4	
12	1 + 2 + 5 + 13	**D_8**	**2^2**	3∩4	
13	1 + 2 + 5 + 12	**Q_8**	**2^2**	2∩5	
14	1 + 2 + 5 + 14	**Q_8**	**2^2**	3∩5	
16	1 + 2 + 5	**4**	**2^3**	4∩9	**G′**
18	1 + 2	**2**	**D_8 × 2**	4∩8	

Maimal subgroups: [**8 × 2**], [**D_8 × 2**], [**$D_{4,4}$**]

32.28 = ⟨A^4, $B^4 = A^2$, $C^2 = A^2$, $CA = A^{-1}C$, $CB = ABC$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	A^2	AB^2	A^3B^2	A A^3	B^2 A^2B^2	B AB	A^2B A^3B	AB^3 A^2B^3	B^3 A^3B^3	A^nC	A^nB^2C	A^nBC	A^nB^3C

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	
#	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	1	–1	i	i	–i	–i	1	–1	i	–i	4
χ_6	1	1	–1	–1	1	–1	i	i	–i	–i	–1	1	–i	i	5
χ_7	1	1	–1	–1	1	–1	–i	–i	i	i	1	–1	–i	i	4
χ_8	1	1	–1	–1	1	–1	–i	–i	i	i	–1	1	i	–i	5
χ_9	2	2	2	2	–2	–2	0	0	0	0	0	0	0	0	6
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	7
χ_{11}	2	–2	2	–2	0	0	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0	0	0	8
χ_{12}	2	–2	2	–2	0	0	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	0	0	0	0	8
χ_{13}	2	–2	–2	2	0	0	$\sqrt{2}$i	$-\sqrt{2}$i	$-\sqrt{2}$i	$\sqrt{2}$i	0	0	0	0	9
χ_{14}	2	–2	–2	2	0	0	$-\sqrt{2}$i	$\sqrt{2}$i	$\sqrt{2}$i	$-\sqrt{2}$i	0	0	0	0	9
^	**1**	**2**	**2**	**2**	**4**	**4**	**8**	**8**	**8**	**8**	**4**	**4**	**4**	**4**	

	Classes	**H**	**G/H**	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	**8 × 2**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	**Q_8 × 2**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	**$D_{4,4}$**	**2**	3	
4	1 + 2 + 5 + 11	**Q_8**	**4**	4	
5	1 + 2 + 5 + 12	**Q_8**	**4**	5	
6	1 + 2 + 3 + 4	**2^2**	**D_8**	6	**Z = $\mathfrak{A}$**
7	1 + 2 + 6	**2**	**16.09**	7	
8	1 + 3	**2**	**Q_{16}**	8	
9	1 + 4	**2**	**Q_{16}**	9	
10	1 + 2 + 3 + 4 + 5 + 6	**4 × 2**	**2^2**	1∩2	**Z_2 = Φ**
11	1 + 2 + 5	**4**	**4 × 2**	1∩4	**G′**
12	1 + 2	**4**	**D_8**	4∩6	

Maximal subgroups: [**8 × 2**], [**Q_8 × 2**], [**$D_{4,4}$**]

32.29 = $D_{8,4}$ = $\langle A^8, B^4, BA = A^{-1}B \rangle$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^4	B^2	A^4B^2	A^2 A^6	A^2B^2 A^6B^2	A A^7	A^3 A^5	AB^2 A^7B^2	A^3B^2 A^5B^2	$A^{2n}B$	$A^{2n}B^3$	$A^{2n+1}B$	$A^{2n+1}B^3$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	K
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	1	–1	1	1	–1	–1	i	–i	i	–i	4
χ_6	1	1	–1	–1	1	–1	1	1	–1	–1	–i	i	–i	i	4
χ_7	1	1	–1	–1	1	–1	–1	–1	1	1	i	–i	–i	i	5
χ_8	1	1	–1	–1	1	–1	–1	–1	1	1	–i	i	i	–i	5
χ_9	2	2	2	2	–2	–2	0	0	0	0	0	0	0	0	6
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	7
χ_{11}	2	–2	2	–2	0	0	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0	0	0	8
χ_{12}	2	–2	2	–2	0	0	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	0	0	0	0	8
χ_{13}	2	–2	–2	2	0	0	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	0	0	0	0	9
χ_{14}	2	–2	–2	2	0	0	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0	0	0	9
^	1	2	2	2	4	4	8	8	8	8	4	4	4	4	

Normal subgroups

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	**8 × 2**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	**$D_{4,4}$**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	**$D_{4,4}$**	**2**	3	
4	1 + 2 + 5 + 7 + 8	**8**	**4**	4	
5	1 + 2 + 5 + 9 + 10	**8**	**4**	5	
6	1 + 2 + 3 + 4	**2^2**	**D_8**	6	**Z = N**
7	1 + 2 + 6	**4**	**Q_8**	7	
8	1 + 3	**2**	**D_{16}**	8	
9	1 + 4	**2**	**Q_{16}**	9	
10	1 + 2 + 3 + 4 + 5 + 6	**4 × 2**	**2^2**	1∩2	**Z_2 = Φ**
11	1 + 2 + 5	**4**	**4 × 2**	2∩4	**G′**
12	1 + 2	**2**	**$D_{4,4}$**	4∩6	

Maximal subgroups: [**8 × 2**], [**$D_{4,4}$**] × 2

32.30 = $M_{8,4}^{(3)} = \langle A^8, B^4, BA = A^3B \rangle$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^4	B^2	A^4B^2	A^2 A^6	A^2B^2 A^6B^2	A A^3	A^5 A^7	AB^2 A^3B^2	A^5B^2 A^7B^2	$A^{2n}B$	$A^{2n}B^3$	$A^{2n+1}B$	$A^{2n+1}B^3$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	1	–1	1	1	–1	–1	i	–i	i	–i	4
χ_6	1	1	–1	–1	1	–1	1	1	–1	–1	–i	i	–i	I	4
χ_7	1	1	–1	–1	1	–1	–1	–1	1	1	i	–i	–i	I	5
χ_8	1	1	–1	–1	1	–1	–1	–1	1	1	–i	i	i	–i	5
χ_9	2	2	2	2	–2	–2	0	0	0	0	0	0	0	0	6
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	7
χ_{11}	2	–2	2	–2	0	0	$\sqrt{2}i$	$-\sqrt{2}i$	$\sqrt{2}i$	$-\sqrt{2}i$	0	0	0	0	8
χ_{12}	2	–2	2	–2	0	0	$-\sqrt{2}i$	$\sqrt{2}i$	$-\sqrt{2}i$	$\sqrt{2}i$	0	0	0	0	8
χ_{13}	2	–2	–2	2	0	0	$\sqrt{2}i$	$-\sqrt{2}i$	$-\sqrt{2}i$	$\sqrt{2}i$	0	0	0	0	9
χ_{14}	2	–2	–2	2	0	0	$-\sqrt{2}i$	$\sqrt{2}i$	$\sqrt{2}i$	$-\sqrt{2}i$	0	0	0	0	9
^	1	2	2	2	4	4	8	8	8	8	4	4	4	4	

Normal subgroups

	Classes	H	G/H	K	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	**8 × 2**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{D_{4,4}}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	$\mathbf{D_{4,4}}$	**2**	3	
4	1 + 2 + 5 + 7 + 8	**8**	**4**	4	
5	1 + 2 + 5 + 9			5	
6	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{D_8}$	6	**Z = $\mathfrak{Z}$**
7	1 + 2 + 6	**4**	$\mathbf{Q_8}$	7	
8	1 + 3	**2**	$\mathbf{M_{8,2}^{(3)}}$	8	
9	1 + 4	**2**	$\mathbf{M_{8,2}^{(3)}}$	9	
10	1 + 2 + 3 + 4 + 5 + 6	**4 × 2**	$\mathbf{2^2}$	1∩2	$\mathbf{Z_2 = \Phi}$
11	1 + 2 + 5	**4**	**4 × 2**	2∩4	**G′**
12	1 + 2	**2**	$\mathbf{D_{4,4}}$	4∩6	

Maximal subgroups: [**8 × 2**], [$\mathbf{D_{4,4}}$] × 2

32.31 = ⟨A^4, B^4, C^2, $CB = AB^{-1}C$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^2	A	A^3	B AB^3	B^2 A^2B^2	B^3 A^3B	AB A^2B^3	A^2B A^3B^3	AB^2 A^3B^2	C ABC A^2C A^3B^2C	AC A^2BC A^3C B^2C	BC AB^3C A^2BC A^3B^3C	ABC A^2B^3C A^3BC B^3C

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	–1	1	–1	–1	–1	1	1	1	–1	–1	1
χ_3	1	1	–1	–1	i	–1	–i	–i	i	1	1	–1	i	–i	2
χ_4	1	1	–1	–1	–i	–1	i	i	–i	1	1	–1	–i	i	2
χ_5	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	3
χ_6	1	1	1	1	–1	1	–1	–1	–1	1	–1	–1	1	1	4
χ_7	1	1	–1	–1	i	–1	–i	–i	i	1	–1	1	–i	i	5
χ_8	1	1	–1	–1	–i	–1	i	i	–i	1	–1	1	i	–i	5
χ_9	2	2	–2	–2	0	2	0	0	0	–2	0	0	0	0	6
χ_{10}	2	2	2	2	0	–2	0	0	0	–2	0	0	0	0	7
χ_{11}	2	–2	2i	–2i	1+i	0	1–i	–1+i	–1–i	0	0	0	0	0	0
χ_{12}	2	–2	–2i	2i	1–i	0	1+i	–1–i	–1+i	0	0	0	0	0	0
χ_{13}	2	–2	2i	–2i	–1–i	0	–1+i	1–i	1+i	0	0	0	0	0	0
χ_{14}	2	–2	–2i	2i	–1+i	0	–1–i	1+i	1–i	0	0	0	0	0	0
^	1	2	4	4	4	2	4	4	4	4	2	4	8	8	

Normal subgroups

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 6 + 10 + 11 + 12	16.08	2	1	
2	1 + 2 + 10 + 11	$\mathbf{D}_8$	4	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	4^2	2	3	
4	1 + 2 + 3 + 4 + 6 + 10 + 13 + 14	$\mathbf{M}_{8,2}^{(5)}$	2	4	
5	1 + 2 + 10 + 12	$\mathbf{Q}_8$	4	5	
6	1 + 2 + 6	2^2	$\mathbf{D}_8$	6	
7	1 + 2 + 3 + 4	4	$\mathbf{D}_8$	7	$\mathbf{Z} = \mathfrak{Z}$
8	1 + 2 + 3 + 4 + 6 + 10	4 × 2	2^2	1∩3	$\mathbf{Z}_2 = \mathbf{\Phi}$
9	1 + 2 + 10	4	4 × 2	2∩3	$\mathbf{G}'$
10	1 + 2	2	16.09	2∩6	

Maximal subgroups: [$\mathbf{4^2}$], [**16.08**], [$\mathbf{M}_{8,2}^{(5)}$]

32.32 = ⟨A^8, $B^4 = A^4$, $BA = A^3B$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^4	B^2	A^4B^2	A^2 A^6	A^2B^2 A^6B^2	A^5B^2 A^7B^2	AB^2 A^3B^2	A A^3	A^5 A^7	$A^{2n}B$	$A^{2n}B^5$	$A^{2n+1}B^3$	$A^{2n+1}B$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	–1	–1	1	–1	1	1	–1	–1	i	–i	i	–i	4
χ_6	1	1	–1	–1	1	–1	1	1	–1	–1	–i	i	–i	i	4
χ_7	1	1	–1	–1	1	–1	–1	–1	1	1	i	–i	–i	i	5
χ_8	1	1	–1	–1	1	–1	–1	–1	1	1	–i	i	i	–i	5
χ_9	2	2	2	2	–2	–2	0	0	0	0	0	0	0	0	6
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	7
χ_{11}	2	–2	2i	–2i	0	0	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}i$	$-\sqrt{2}i$	0	0	0	0	0
χ_{12}	2	–2	2i	–2i	0	0	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}i$	$\sqrt{2}i$	0	0	0	0	0
χ_{13}	2	–2	–2i	2i	0	0	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}i$	$\sqrt{2}i$	0	0	0	0	0
χ_{14}	2	–2	–2i	2i	0	0	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}i$	$-\sqrt{2}i$	0	0	0	0	0
^	1	2	4	4	4	2	8	8	8	8	8	8	8	8	

Normal subgroups

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	8×2	2	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{M}_{8,2}^{(5)}$	2	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	$\mathbf{M}_{8,2}^{(5)}$	2	3	
4	1 + 2 + 5 + 7 + 8	8	4	4	
5	1 + 2 + 5 + 9 + 10	8	4	5	
6	1 + 2 + 3 + 4	4	$\mathbf{D}_8$	6	$\mathbf{Z} = \mathfrak{Z}$
7	1 + 2 + 6	2^2	$\mathbf{Q}_8$	7	
8	1 + 2 + 3 + 4 + 5 + 6	4×2	2^2	1∩2	$\mathbf{Z}_2 = \mathbf{\Phi}$
9	1 + 2 + 5	4	4×2	3∩4	$\mathbf{G}'$
10	1 + 2	2	$\mathbf{D}_{4,4}$	4∩6	

Mazimal subgroups: [4^2], [$\mathbf{M}_{8,2}^{(5)}$] × 2

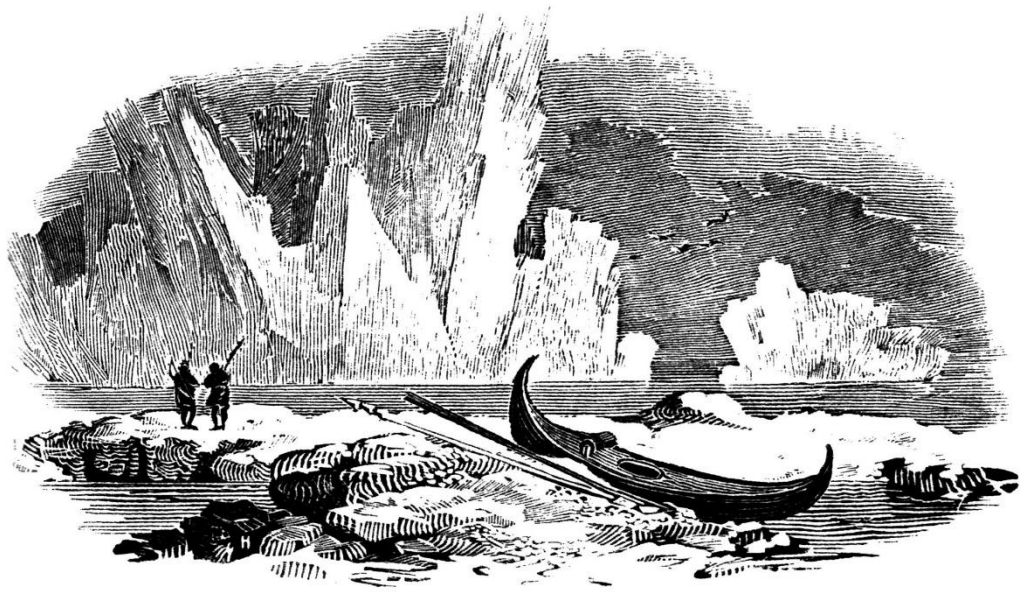
32.33 = ⟨A^2, B^2, C^2, D^2, E^2, EC = ACE, ED = BDE⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A	B	AB	C AC	BC ABC	D BD	AD ABD	CD ABCD	ACD BCD	A^mB^nE	A^mB^nCE	A^mB^nDE	A^mB^nCDE

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	–2	2	–2	0	0	2	–2	0	0	0	0	0	0	8
χ_{10}	2	–2	2	–2	0	0	–2	2	0	0	0	0	0	0	9
χ_{11}	2	–2	–2	2	0	0	0	0	2	–2	0	0	0	0	10
χ_{12}	2	–2	–2	2	0	0	0	0	–2	2	0	0	0	0	11
χ_{13}	2	2	–2	–2	2	–2	0	0	0	0	0	0	0	0	12
χ_{14}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	13
^	1	2	2	2	2	2	2	2	2	2	2	4	4	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{2^4}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{D_8 \times 2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	**16.09**	**2**	3	
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	$\mathbf{D_8 \times 2}$	**2**	4	
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	**16.09**	**2**	5	
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	$\mathbf{D_8 \times 2}$	**2**	6	
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	**16.09**	**2**	7	
8	1 + 3 + 7	$\mathbf{2^2}$	$\mathbf{D_8}$	8	
9	1 + 3 + 8	$\mathbf{2^2}$	$\mathbf{D_8}$	9	
10	1 + 4 + 9	$\mathbf{2^2}$	$\mathbf{D_8}$	10	
11	1 + 4 + 10	$\mathbf{2^2}$	$\mathbf{D_8}$	11	
12	1 + 2 + 5	$\mathbf{2^2}$	$\mathbf{D_8}$	12	
13	1 + 2 + 6	$\mathbf{2^2}$	$\mathbf{D_8}$	13	
14	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{2^3}$	$\mathbf{2^2}$	1∩2	
15	1 + 2 + 3 + 4 + 7 + 8	$\mathbf{2^3}$	$\mathbf{2^2}$	1∩4	
16	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{2^3}$	$\mathbf{2^2}$	1∩6	
17	1 + 2 + 3 + 4 + 11	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩4	
18	1 + 2 + 3 + 4 + 12	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩5	
19	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{2^3}$	2∩7	$\mathbf{G' = Z = \Phi = \mathfrak{R}}$
20	1 + 4	**2**	$\mathbf{D_8 \times 2}$	2∩10	
21	1 + 2 + 3 + 4 + 13	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩4	
22	1 + 2 + 3 + 4 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩5	
23	1 + 3	**2**	$\mathbf{D_8 \times 2}$	8∩9	
24	1 + 2	**2**	$\mathbf{D_8 \times 2}$	12∩13	

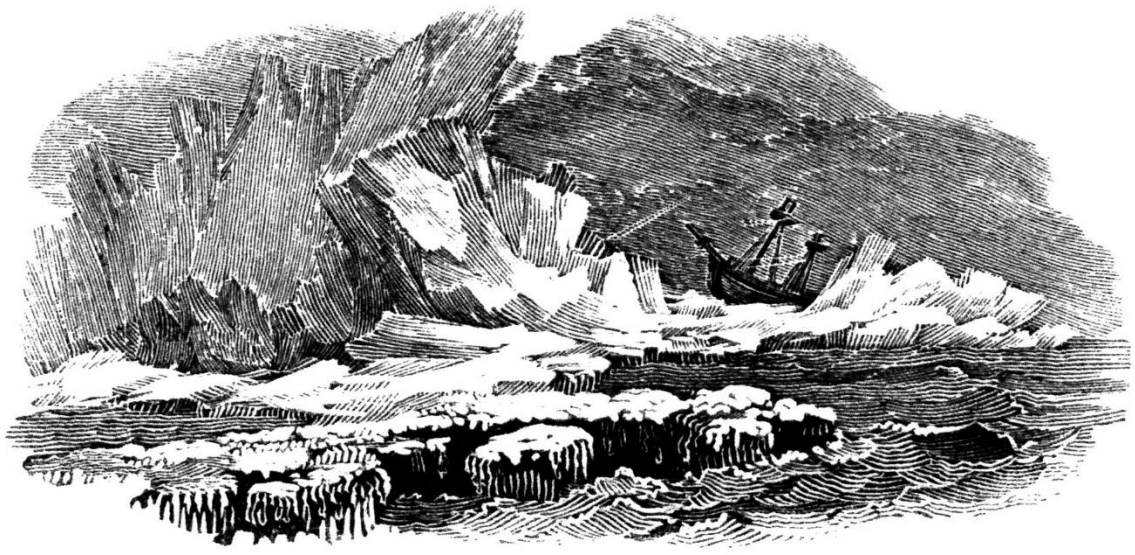
Maximal subgroups: [$\mathbf{2^4}$], [$\mathbf{D_8 \times 2}$] × 3, [**16.09**] × 3



32.34 = ⟨A^4, B^4, C^2, $CA = A^{-1}C$, $CB = B^{-1}C$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^2	B^2	A^2B^2	A A^3	AB^2 A^3B^2	B B^3	A^2B A^2B^3	AB A^3B^3	A^3B A^3B^3	C A^2C B^2C A^2B^2C	AC A^3C AB^2C A^3B^2C	BC B^3C A^2BC A^3B^3C	ABC AB^3C A^3BC A^3B^3C

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	–2	–2	2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2	–2	0	0	2	–2	0	0	0	0	0	0	10
χ_{12}	2	–2	2	–2	0	0	–2	2	0	0	0	0	0	0	11
χ_{13}	2	–2	–2	2	0	0	0	0	2	–2	0	0	0	0	12
χ_{14}	2	–2	–2	2	0	0	0	0	–2	2	0	0	0	0	13
^	1	2	2	2	4	4	4	4	4	4	2	2	2	2	



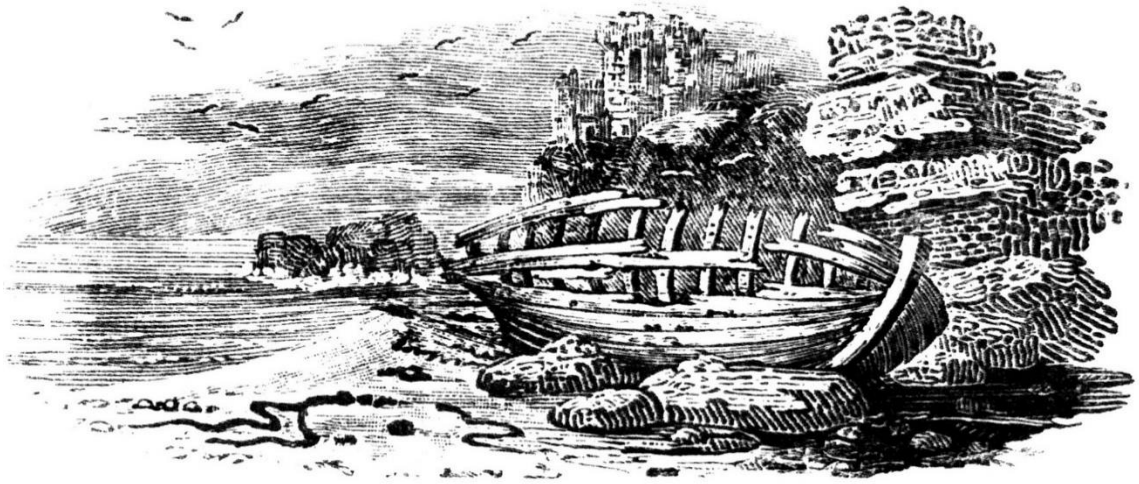
	Classes	H	G/H	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{4^2}$	**2**	1
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{D_8 \times 2}$	**2**	2
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	$\mathbf{D_8 \times 2}$	**2**	3
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	$\mathbf{D_8 \times 2}$	**2**	4
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	$\mathbf{D_8 \times 2}$	**2**	5
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	$\mathbf{D_8 \times 2}$	**2**	6
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	$\mathbf{D_8 \times 2}$	**2**	7
8	1 + 2 + 5	**4**	$\mathbf{D_8}$	8
9	1 + 2 + 6	**4**	$\mathbf{D_8}$	9
10	1 + 3 + 7	**4**	$\mathbf{D_8}$	10
11	1 + 3 + 8	**4**	$\mathbf{D_8}$	11
12	1 + 4 + 9	**4**	$\mathbf{D_8}$	12
13	1 + 4 + 10	**4**	$\mathbf{D_8}$	13
14	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩2
15	1 + 2 + 3 + 4 + 7 + 8	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩4
16	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩6
17	1 + 2 + 3 + 4 + 11	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩4
18	1 + 2 + 3 + 4 + 12	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩5
19	1 + 3	**2**	$\mathbf{D_8 \times 2}$	2∩10
20	1 + 4	**2**	$\mathbf{D_8 \times 2}$	2∩12
21	1 + 2 + 3 + 4 + 14	$\mathbf{2^3}$	$\mathbf{2^2}$	3∩5
22	1 + 2 + 3 + 4 + 13	$\mathbf{2^3}$	$\mathbf{2^2}$	3∩7
23	1 + 2	**2**	$\mathbf{D_8 \times 2}$	4∩8
24	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{2^3}$	14∩16

Maximal subgroups: $[\mathbf{4^2}]$, $[\mathbf{D_8 \times 2}] \times 6$

32.35 = ⟨A^4, B^4, $C^2 = A^2$, $CA = A^{-1}C$, $CB = B^{-1}C$⟩

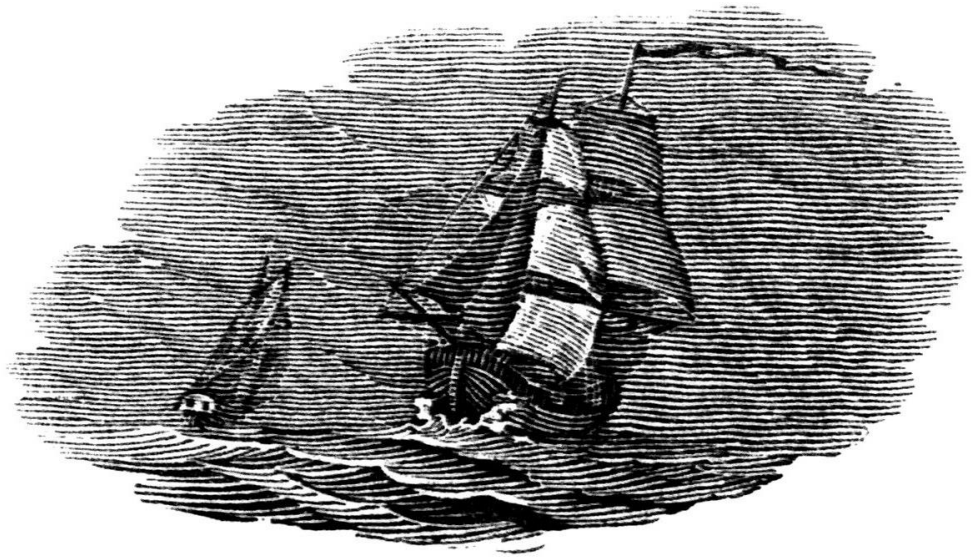
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	A^2	B^2	A^2B^2	A A^3	AB^2 A^3B^2	B B^3	A^2B A^2B^3	AB A^3B^3	AB^3 A^3B	C A^2C B^2C A^2B^2C	AC AB^2C A^3C A^3B^2C	BC B^3C A^2BC A^2B^3C	ABC AB^3C A^3BC A^3B^3C

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	
#	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	–2	–2	2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2	–2	0	0	2	–2	0	0	0	0	0	0	10
χ_{12}	2	–2	2	–2	0	0	–2	2	0	0	0	0	0	0	11
χ_{13}	2	–2	–2	2	0	0	0	0	2	–2	0	0	0	0	12
χ_{14}	2	–2	–2	2	0	0	0	0	–2	2	0	0	0	0	13
^	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	



	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{4^2}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{Q_8 \times 2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	$\mathbf{Q_8 \times 2}$	**2**	3	
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	$\mathbf{Q_8 \times 2}$	**2**	4	
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	$\mathbf{Q_8 \times 2}$	**2**	5	
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	$\mathbf{D_{4,4}}$	**2**	6	
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	$\mathbf{D_{4,4}}$	**2**	7	
8	1 + 2 + 5	**4**	$\mathbf{D_8}$	8	
9	1 + 2 + 6	**4**	$\mathbf{D_8}$	9	
10	1 + 3 + 7	**4**	$\mathbf{Q_8}$	10	
11	1 + 3 + 8	**4**	$\mathbf{Q_8}$	11	
12	1 + 4 + 9	**4**	$\mathbf{Q_8}$	12	
13	1 + 4 + 10	**4**	$\mathbf{Q_8}$	13	
14	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩2	$\mathfrak{N}$
15	1 + 2 + 3 + 4 + 7 + 8	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩4	
16	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩6	
17	1 + 2 + 3 + 4 + 11	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩4	
18	1 + 2 + 3 + 4 + 12	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩5	
19	1 + 3	**2**	$\mathbf{Q_8 \times 2}$	2∩10	
20	1 + 4	**2**	$\mathbf{Q_8 \times 2}$	2∩12	
21	1 + 2 + 3 + 4 + 13	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩4	
22	1 + 2 + 3 + 4 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩5	
23	1 + 2	**2**	$\mathbf{D_8 \times 2}$	4∩8	
24	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{2^3}$	14∩15	**G′ = Z = Φ**

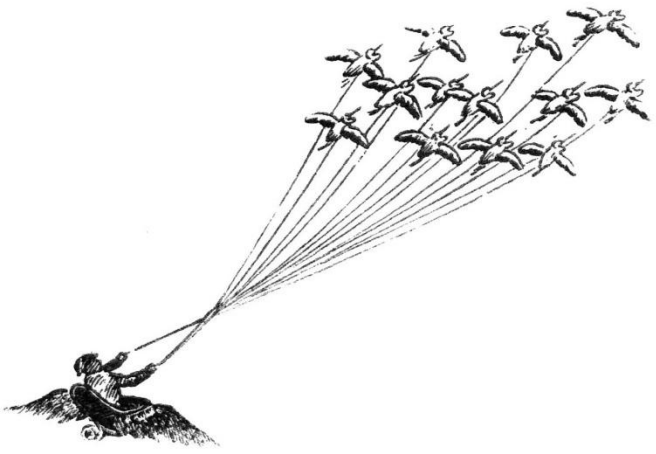
Maximal subgroups: [$\mathbf{4^2}$], [$\mathbf{Q_8 \times 2}$] × 2, [$\mathbf{D_{4,4}}$] × 4



32.36 = ⟨A^4, B^2, C^2, D^2, $DA = A^{-1}D$, $DC = BCD$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^2	B	A^2B	A A^3	AB A^3B	C BC	A^2C A^2BC	AC A^3BC	A^3C ABC	D A^2D BD A^2BD	AD ABD A^2D A^2BD	CD BCD A^2CD A^2BCD	ACD ABCD A^3CD A^3BCD

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	–2	–2	2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2	–2	0	0	2	–2	0	0	0	0	0	0	10
χ_{12}	2	–2	2	–2	0	0	–2	2	0	0	0	0	0	0	11
χ_{13}	2	–2	–2	2	0	0	0	0	2i	–2i	0	0	0	0	12
χ_{14}	2	–2	–2	2	0	0	0	0	–2i	2i	0	0	0	0	12
^	1	2	2	2	4	4	2	2	4	4	2	2	4	4	



	Classes	H	G/H	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	**4 × 2²**	**2**	1
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	**D₈ × 2**	**2**	2
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	**D₄,₄**	**2**	3
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	**D₈ × 2**	**2**	4
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	**D₈ × 2**	**2**	5
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	**16.09**	**2**	6
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	**16.09**	**2**	7
8	1 + 2 + 5	**4**	**D₈**	8
9	1 + 2 + 6	**4**	**D₈**	9
10	1 + 3 + 7	**2²**	**D₈**	10
11	1 + 3 + 8	**2²**	**D₈**	11
12	1 + 4	**2**	**16.08**	12
13	1 + 2 + 3 + 4 + 5 + 6	**4 × 2**	**2²**	1∩2
14	1 + 2 + 3 + 4 + 7 + 8	**2³**	**2²**	1∩4
15	1 + 2 + 3 + 4 + 9 + 10	**4 × 2**	**2²**	1∩6
16	1 + 2 + 3 + 4 + 11	**2³**	**2²**	2∩4
17	1 + 2 + 3 + 4 + 12	**2³**	**2²**	2∩5
18	1 + 3	**2**	**D₈ × 2**	2∩10
19	1 + 2 + 3 + 4 + 13	**4 × 2**	**2²**	3∩4
20	1 + 2 + 3 + 4 + 14	**4 × 2**	**2²**	3∩5
21	1 + 2	**2**	**D₈ × 2**	4∩8
22	1 + 2 + 3 + 4	**2²**	**2³**	12∩13

G′ = Z = Φ = 𝔄

Maximal subgroups: [**4 × 2²**] × 2, [**D₈ × 2**] × 3, [**16.09**] × 2, [**D₄,₄**]

32.37 = ⟨A^4, $B^2 = A^2$, C^2, D^2, $BA = A^{-1}B$, $DB = BCD$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^2	C	A^2C	A A^3	AC A^3C	D CD	A^2D A^2CD	AD A^3CD	A^3D ACD	B BC A^2B A^2BC	AB ABC A^3B A^3BC	BD BCD A^2BD A^2BCD	ABD ABCD A^3BD A^3BCD

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	–2	–2	2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2	–2	0	0	2	–2	0	0	0	0	0	0	10
χ_{12}	2	–2	2	–2	0	0	–2	2	0	0	0	0	0	0	11
χ_{13}	2	–2	–2	2	0	0	0	0	2i	–2i	0	0	0	0	12
χ_{14}	2	–2	–2	2	0	0	0	0	–2i	2i	0	0	0	0	12
^	1	2	2	2	4	4	2	2	4	4	4	4	4	4	

	Classes	H	G/H	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{4 \times 2^2}$	**2**	1
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{Q_8 \times 2}$	**2**	2
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	$\mathbf{D_{4,4}}$	**2**	3
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	**16.09**	**2**	4
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	**16.09**	**2**	5
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	$\mathbf{Q_8 \times 2}$	**2**	6
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	$\mathbf{Q_8 \times 2}$	**2**	7
8	1 + 2 + 5	**4**	$\mathbf{D_8}$	8
9	1 + 2 + 6	**4**	$\mathbf{D_8}$	9
10	1 + 3 + 7	$\mathbf{2^2}$	$\mathbf{Q_8}$	10
11	1 + 3 + 8	$\mathbf{2^2}$	$\mathbf{Q_8}$	11
12	1 + 4	**2**	**16.08**	12
13	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩2
14	1 + 2 + 3 + 4 + 7 + 8	$\mathbf{2^3}$	$\mathbf{2^2}$	1∩4
15	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩6
16	1 + 2 + 3 + 4 + 11	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩4
17	1 + 2 + 3 + 4 + 12	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩5
18	1 + 3	**2**	$\mathbf{Q_8 \times 2}$	2∩10
19	1 + 2 + 3 + 4 + 13	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩4
20	1 + 2 + 3 + 4 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩5
21	1 + 2	**2**	$\mathbf{Q_8 \times 2}$	4∩8
22	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{2^3}$	13∩14

$\mathbf{G' = Z = \Phi = \mathfrak{R}}$

Maximal subgroups: [$\mathbf{4 \times 2^2}$], [$\mathbf{D_8 \times 2}$] × 3, [**16.09**] × 2, [$\mathbf{D_{4,4}}$]

32.38 = ⟨A^4, B^2, C^2, D^2, DA = ABD, DC = A^2CD⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^2	B	A^2B	C A^2C	BC A^2BC	A AB	A^3 A^3B	AC A^3BC	ABC A^3C	D BD A^2D A^2BD	CD BCD A^2CD A^2BCD	AD ABD A^3D A^3BD	ACD ABCD A^3CD A^3BCD

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	–2	–2	2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2	–2	0	0	2i	–2i	0	0	0	0	0	0	10
χ_{12}	2	–2	2	–2	0	0	–2i	2i	0	0	0	0	0	0	10
χ_{13}	2	–2	–2	2	0	0	0	0	2i	–2i	0	0	0	0	11
χ_{14}	2	–2	–2	2	0	0	0	0	–2i	2i	0	0	0	0	11
^	1	2	2	2	2	2	4	4	4	4	2	4	4	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{4 \times 2^2}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{D_8 \times 2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	**16.09**	**2**	3	
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	**16.09**	**2**	4	
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	$\mathbf{D_{4,4}}$	**2**	5	
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	**16.09**	**2**	6	
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	$\mathbf{D_{4,4}}$	**2**	7	
8	1 + 2 + 5	$\mathbf{2^2}$	$\mathbf{D_8}$	8	
9	1 + 2 + 6	$\mathbf{2^2}$	$\mathbf{D_8}$	9	
10	1 + 3	**2**	**16.08**	10	
11	1 + 4	**2**	**16.08**	11	
12	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{2^3}$	$\mathbf{2^2}$	1∩2	
13	1 + 2 + 3 + 4 + 7 + 8	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩4	
14	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩6	
15	1 + 2 + 3 + 4 + 11	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩4	
16	1 + 2 + 3 + 4 + 12	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩5	
17	1 + 2 + 3 + 4 + 13	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩4	
18	1 + 2 + 3 + 4 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩5	
19	1 + 2	**2**	$\mathbf{D_8 \times 2}$	4∩8	
20	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{2^3}$	12∩13	$\mathbf{G' = Z = \Phi = \mathfrak{N}}$

Maximal subgroups: [$\mathbf{4 \times 2^2}$], [$\mathbf{D_8 \cap 2}$], [**16.09**] ∩ 3, [$\mathbf{D_{4,4}}$] ∩ 2

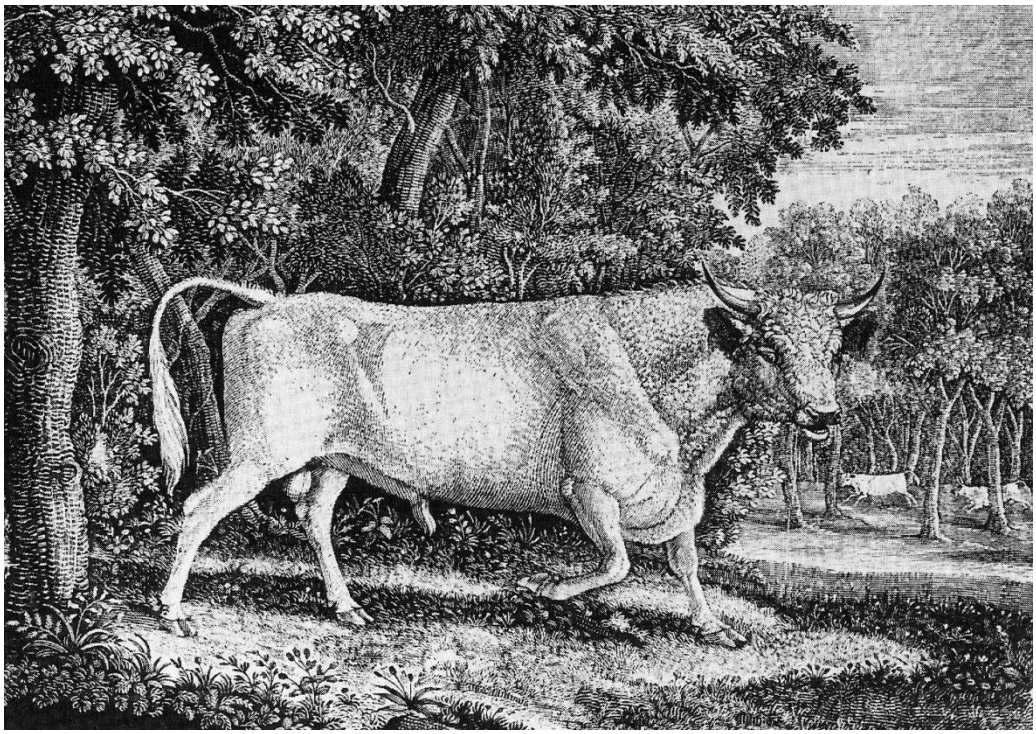
32.39 = ⟨A^4, B^4, C^2, CA = $A^{-1}C$, CB = $B^{-1}C$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	A^2	B^2	A^2B^2	A A^3	AB^2 A^3B^2	B B^3	A^2B A^2B^3	AB A^3B^3	A^3B AB^3	C A^2C B^2C A^2B^2C	AC AB^2C A^3C A^3B^2C	BC B^3C A^2BC A^2B^3C	ABC AB^3C A^3BC A^3B^3C

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	
#	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	–2	–2	2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2	–2	0	0	2i	–2i	0	0	0	0	0	0	10
χ_{12}	2	–2	2	–2	0	0	–2i	2i	0	0	0	0	0	0	10
χ_{13}	2	–2	–2	2	0	0	0	0	2i	–2i	0	0	0	0	11
χ_{14}	2	–2	–2	2	0	0	0	0	–2i	2i	0	0	0	0	11
^	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**4**	**4**	**2**	**2**	**4**	**4**	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{4^2}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{D_8 \times 2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	**16**	**2**	3	
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	**16.09**	**2**	4	
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	**16.09**	**2**	5	
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	**16.09**	**2**	6	
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	**16.09**	**2**	7	
8	1 + 2 + 5	**4**	$\mathbf{D_8}$	8	
9	1 + 2 + 6	**4**	$\mathbf{D_8}$	9	
10	1 + 3	**2**	$\mathbf{2^4}$	10	
11	1 + 4	**2**	$\mathbf{D_8 \times 2}$	11	
12	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩2	
13	1 + 2 + 3 + 4 + 7 + 8	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩4	
14	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩6	
15	1 + 2 + 3 + 4 + 11	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩4	
16	1 + 2 + 3 + 4 + 12	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩5	
17	1 + 2 + 3 + 4 + 13	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩4	
18	1 + 2 + 3 + 4 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩5	
19	1 + 2	**2**	$\mathbf{D_8 \times 2}$	4∩8	
20	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{2^3}$	12∩13	$\mathbf{G' = Z = \Phi = N}$

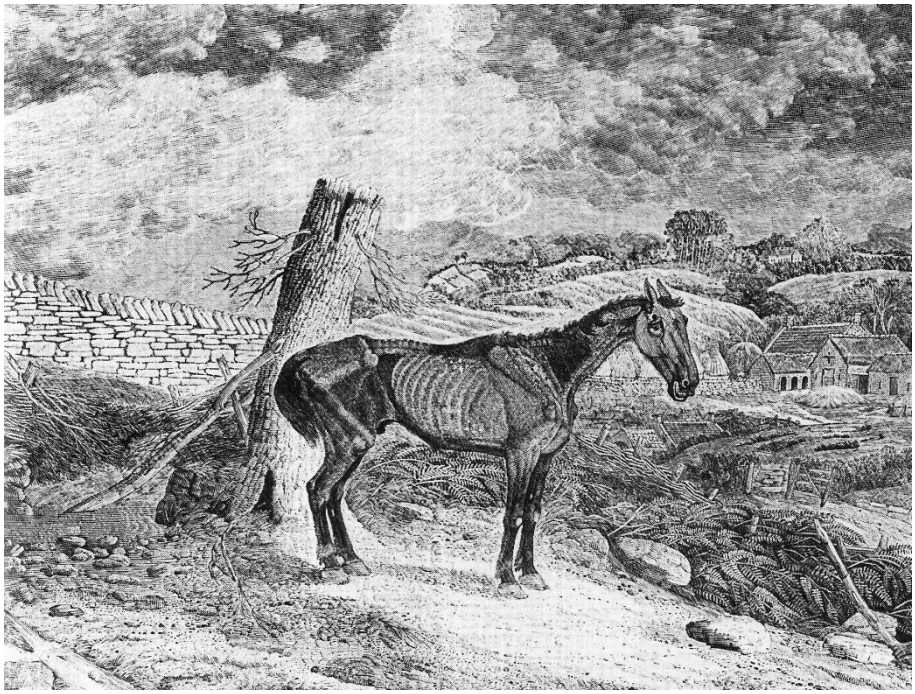
Maximal subgroups: [$\mathbf{4^2}$], [$\mathbf{D_8 \times 2}$], [$\mathbf{Q_8 \times 2}$], [**16.09**] × 4



32.40 = ⟨A^4, B^4, $C^2 = A^2B^2$, $BA = A^{-1}B$, $CB = B^{-1}C$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	A^2	B^2	A^2B^2	A A^3	AB^2 A^3B^2	C B^2C	A^2C A^2B^2C	AC A^3B^2C	A^3C AB^2C	B B^3 A^2B A^2B^3	AB A^3B AB^3 A^3B^3	BC B^3C A^2BC A^2B^3C	ABC AB^3C A^3BC A^3B^3C

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	1	1	1	2	2	2	2	2	2	4	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	–2	–2	2	–2	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2	2	0	0	0	0	0	0	0	0	9
χ_{11}	2	–2	2	–2	0	0	2i	–2i	0	0	0	0	0	0	10
χ_{12}	2	–2	2	–2	0	0	–2i	2i	0	0	0	0	0	0	10
χ_{13}	2	–2	–2	2	0	0	0	0	2i	–2i	0	0	0	0	11
χ_{14}	2	–2	–2	2	0	0	0	0	–2i	2i	0	0	0	0	11
^	1	2	2	2	4	4	4	4	4	4	2	2	4	4	



	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{D_{4,4}}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	$\mathbf{D_8 \times 2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	$\mathbf{D_{4,4}}$	**2**	3	
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	**16.08**	**2**	4	
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	$\mathbf{4 \times 2^2}$	**2**	5	
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	$\mathbf{4 \times 2^2}$	**2**	6	
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	$\mathbf{4 \times 2^2}$	**2**	7	
8	1 + 2 + 5	**4**	$\mathbf{Q_8}$	8	
9	1 + 2 + 6	**4**	$\mathbf{Q_8}$	9	
10	1 + 3	**2**	**16.08**	10	
11	1 + 4	**2**	**16.08**	11	
12	1 + 2 + 3 + 4 + 5 + 6	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩2	
13	1 + 2 + 3 + 4 + 7 + 8	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩4	
14	1 + 2 + 3 + 4 + 9 + 10	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩6	
15	1 + 2 + 3 + 4 + 11	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩4	
16	1 + 2 + 3 + 4 + 12	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩5	
17	1 + 2 + 3 + 4 + 13	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩4	
18	1 + 2 + 3 + 4 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩5	
19	1 + 2	**2**	$\mathbf{Q_8 \times 2}$	4∩8	
20	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{2^3}$	12∩13	$\mathbf{G' = Z = \Phi = \mathfrak{N}}$

Maximal subgroups: $[\mathbf{4^2}]$, $[\mathbf{D_{4,4}}] \times 6$

32.41 = ⟨A^4, B^4, C^2, $CA = A^{-1}B^2C$, $CB = A^2BC$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	A^2B^2	A^2	B^2	A A^3B^2	A^3 A^3B^2	B AB	B^3 A^2B^3	AB AB^3	A^3B A^3B^3	C A^2C B^2C A^2B^2C	AC AB^2C A^3C A^3B^2C	BC B^3C A^2BC A^2B^3C	ABC AB^3C A^3BC A^3B^3C

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	
#	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	1	–1	–1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	1	–1	–1	1	1	–1	–1	1	–1	1	–1	4
χ_6	1	1	1	1	–1	–1	1	1	–1	–1	–1	1	–1	1	5
χ_7	1	1	1	1	–1	–1	–1	–1	1	1	1	–1	–1	1	6
χ_8	1	1	1	1	–1	–1	–1	–1	1	1	–1	1	1	–1	7
χ_9	2	2	–2	–2	2i	–2i	0	0	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	–2i	2i	0	0	0	0	0	0	0	0	8
χ_{11}	2	–2	2	–2	0	0	2i	–2i	0	0	0	0	0	0	9
χ_{12}	2	–2	2	–2	0	0	–2i	2i	0	0	0	0	0	0	9
χ_{13}	2	–2	–2	2	0	0	0	0	2i	–2i	0	0	0	0	10
χ_{14}	2	–2	–2	2	0	0	0	0	–2i	2i	0	0	0	0	10
^	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**4**	**4**	**2**	**4**	**4**	**4**	

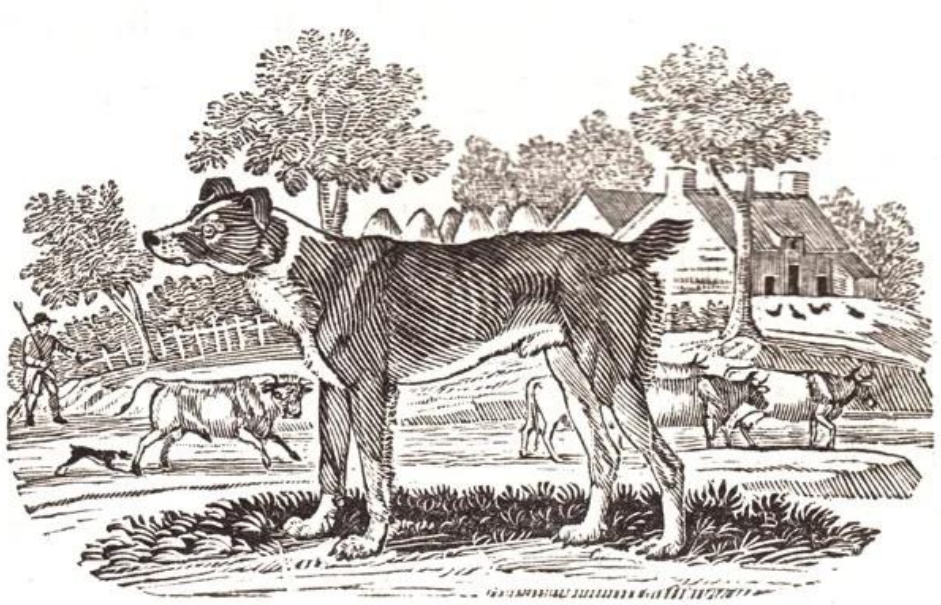
	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	$\mathbf{4^2}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 11 + 12	**16.09**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 13 + 14	$\mathbf{D_{4,4}}$	**2**	3	
4	1 + 2 + 3 + 4 + 7 + 8 + 11 + 13	**16.09**	**2**	4	
5	1 + 2 + 3 + 4 + 7 + 8 + 12 + 14	$\mathbf{D_{4,4}}$	**2**	5	
6	1 + 2 + 3 + 4 + 9 + 10 + 11 + 14	**16.09**	**2**	6	
7	1 + 2 + 3 + 4 + 9 + 10 + 12 + 13	$\mathbf{D_{4,4}}$	**2**	7	
8	1 + 2	**2**	**16.08**	8	
9	1 + 3	**2**	**16.08**	9	
10	1 + 4	**2**	**16.08**	10	
11	1 + 2 + 3 + 4 + 5 + 6	**4 × 2**	$\mathbf{2^2}$	1∩2	
12	1 + 2 + 3 + 4 + 7 + 8	**4 × 2**	$\mathbf{2^2}$	1∩4	
13	1 + 2 + 3 + 4 + 9 + 10	**4 × 2**	$\mathbf{2^2}$	1∩6	
14	1 + 2 + 3 + 4 + 11	$\mathbf{2^3}$	$\mathbf{2^2}$	2∩4	
15	1 + 2 + 3 + 4 + 12	**4 × 2**	$\mathbf{2^2}$	2∩5	
16	1 + 2 + 3 + 4 + 13	**4 × 2**	$\mathbf{2^2}$	3∩4	
17	1 + 2 + 3 + 4 + 14	**4 × 2**	$\mathbf{2^2}$	3∩5	
18	1 + 2 + 3 + 4	$\mathbf{2^2}$	$\mathbf{2^3}$	2∩9	$\mathbf{G' = Z = \Phi = \mathfrak{R}}$

Maximal subgroups: [$\mathbf{4^2}$], [**16.09**] × 3, [$\mathbf{D_{4,4}}$] × 3

32.42 = ⟨A^4, $B^2 = C^2 = D^2 = A^2$, $CB = A^2BC$, $DA = A^{-1}D$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
1	A^2	A A^3	B A^2B	AB A^3B	C A^2C	AC A^3C	BC A^2BC	ABC A^3BC	D A^2D	AD A^3D	BD A^2BD	ABD A^3BD	CD A^2CD	ACD A^3CD	BCD A^2BC D	ABCD A^2BC D

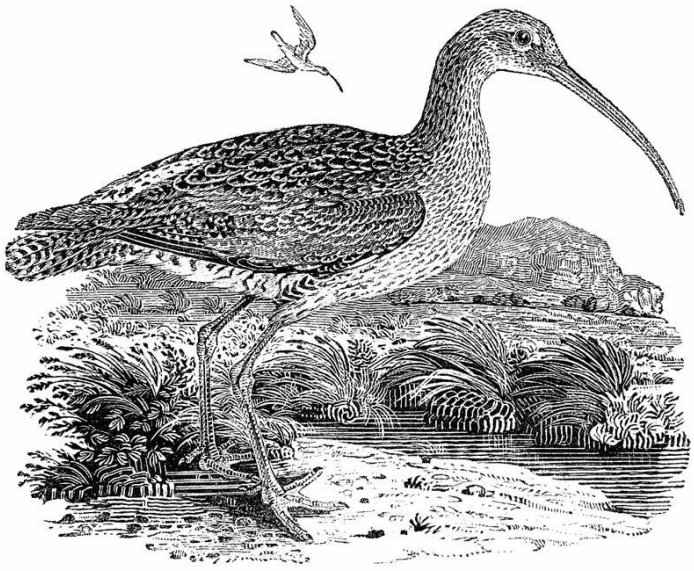
C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
#	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	–1	–1	1	1	1	1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	3
χ_5	1	1	1	–1	–1	1	1	–1	–1	1	1	–1	–1	1	1	–1	–1	4
χ_6	1	1	1	–1	–1	1	1	–1	–1	–1	–1	1	1	–1	–1	1	1	5
χ_7	1	1	1	–1	–1	–1	–1	1	1	1	1	–1	–1	–1	–1	1	1	6
χ_8	1	1	1	–1	–1	–1	–1	1	1	–1	–1	1	1	1	1	–1	–1	7
χ_9	1	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	8
χ_{10}	1	1	–1	1	–1	1	–1	1	–1	–1	1	–1	1	–1	1	–1	1	9
χ_{11}	1	1	–1	1	–1	–1	1	–1	1	1	–1	1	–1	–1	1	–1	1	10
χ_{12}	1	1	–1	1	–1	–1	1	–1	1	–1	1	–1	1	1	–1	1	–1	11
χ_{13}	1	1	–1	–1	1	1	–1	–1	1	1	–1	–1	1	1	–1	–1	1	12
χ_{14}	1	1	–1	–1	1	1	–1	–1	1	–1	1	1	–1	–1	1	1	–1	13
χ_{15}	1	1	–1	–1	1	–1	1	1	–1	1	–1	–1	1	–1	1	1	–1	14
χ_{16}	1	1	–1	–1	1	–1	1	1	–1	–1	1	1	–1	1	–1	–1	1	15
χ_{17}	4	–4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
^	**1**	**2**	**4**	**4**	**2**	**4**	**2**	**4**	**2**	**4**	**4**	**2**	**2**	**2**	**2**	**2**	**2**	



	Classes	H	G/H	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	**16.08**	**2**	1
2	1 + 2 + 3 + 4 + 5 + 10 + 11 + 12 + 13	**16.08**	**2**	2
3	1 + 2 + 3 + 4 + 5 + 14 + 15 + 16 + 17	$\mathbf{D_8 \times 2}$	**2**	3
4	1 + 2 + 3 + 6 + 7 + 10 + 11 + 14 + 15	**16.08**	**2**	4
5	1 + 2 + 3 + 6 + 7 + 12 + 13 + 16 + 17	$\mathbf{D_8 \times 2}$	**2**	5
6	1 + 2 + 3 + 8 + 9 + 10 + 11 + 16 + 17	**16.08**	**2**	6
7	1 + 2 + 3 + 8 + 9 + 12 + 13 + 14 + 15	$\mathbf{D_8 \times 2}$	**2**	7
8	1 + 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16	**16.08**	**2**	8
9	1 + 2 + 4 + 6 + 8 + 11 + 13 + 15 + 17	**16.08**	**2**	9
10	1 + 2 + 4 + 7 + 9 + 10 + 12 + 15 + 17	$\mathbf{D_8 \times 2}$	**2**	10
11	1 + 2 + 4 + 7 + 9 + 11 + 13 + 14 + 16	$\mathbf{D_8 \times 2}$	**2**	11
12	1 + 2 + 5 + 6 + 9 + 10 + 13 + 14 + 17	$\mathbf{D_8 \times 2}$	**2**	12
13	1 + 2 + 5 + 6 + 9 + 11 + 12 + 15 + 16	$\mathbf{D_8 \times 2}$	**2**	13
14	1 + 2 + 5 + 7 + 8 + 10 + 13 + 15 + 16	$\mathbf{D_8 \times 2}$	**2**	14
15	1 + 2 + 5 + 7 + 8 + 11 + 12 + 14 + 17	$\mathbf{D_8 \times 2}$	**2**	15
16	1 + 2 + 3 + 4 + 5	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
17	1 + 2 + 3 + 6 + 7	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
18	1 + 2 + 3 + 8 + 9	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
19	1 + 2 + 4 + 10 + 12	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
20	1 + 2 + 4 + 11 + 13	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
21	1 + 2 + 6 + 10 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
22	1 + 2 + 6 + 11 + 15	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
23	1 + 2 + 8 + 10 + 16	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
24	1 + 2 + 8 + 11 + 17	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	
25	1 + 2 + 5 + 14 + 17	$\mathbf{2^3}$	$\mathbf{2^2}$	
26	1 + 2 + 5 + 15 + 16	$\mathbf{2^3}$	$\mathbf{2^2}$	
27	1 + 2 + 7 + 12 + 17	$\mathbf{2^3}$	$\mathbf{2^2}$	
28	1 + 2 + 7 + 13 + 16	$\mathbf{2^3}$	$\mathbf{2^2}$	
29	1 + 2 + 9 + 12 + 15	$\mathbf{2^3}$	$\mathbf{2^2}$	
30	1 + 2 + 9 + 13 + 14	$\mathbf{2^3}$	$\mathbf{2^2}$	
31	1 + 2 + 3 + 12 + 13	$\mathbf{D_8}$	$\mathbf{2^2}$	
32	1 + 2 + 3 + 14 + 15	$\mathbf{D_8}$	$\mathbf{2^2}$	
33	1 + 2 + 3 + 16 + 17	$\mathbf{D_8}$	$\mathbf{2^2}$	
34	1 + 2 + 4 + 7 + 9	$\mathbf{D_8}$	$\mathbf{2^2}$	
35	1 + 2 + 4 + 14 + 16	$\mathbf{D_8}$	$\mathbf{2^2}$	
36	1 + 2 + 4 + 15 + 17	$\mathbf{D_8}$	$\mathbf{2^2}$	
37	1 + 2 + 5 + 6 + 9	$\mathbf{D_8}$	$\mathbf{2^2}$	
38	1 + 2 + 5 + 7 + 8	$\mathbf{D_8}$	$\mathbf{2^2}$	
39	1 + 2 + 5 + 10 + 13	$\mathbf{D_8}$	$\mathbf{2^2}$	
40	1 + 2 + 5 + 11 +12	$\mathbf{D_8}$	$\mathbf{2^2}$	

41	1 + 2 + 6 + 12 + 16	$\mathbf{D_8}$	$\mathbf{2^2}$	
42	1 + 2 + 6 + 13 + 17	$\mathbf{D_8}$	$\mathbf{2^2}$	
43	1 + 2 + 7 + 10 + 15	$\mathbf{D_8}$	$\mathbf{2^2}$	
44	1 + 2 + 7 + 11 + 14	$\mathbf{D_8}$	$\mathbf{2^2}$	
45	1 + 2 + 8 + 12 + 14	$\mathbf{D_8}$	$\mathbf{2^2}$	
46	1 + 2 + 8 + 13 + 15	$\mathbf{D_8}$	$\mathbf{2^2}$	
47	1 + 2 + 9 + 10 + 17	$\mathbf{D_8}$	$\mathbf{2^2}$	
48	1 + 2 + 9 + 11 + 16	$\mathbf{D_8}$	$\mathbf{2^2}$	
49	1 + 2 + 3 + 10 + 11	$\mathbf{Q_8}$	$\mathbf{2^2}$	
50	1 + 2 + 4 + 6 + 8	$\mathbf{Q_8}$	$\mathbf{2^2}$	
51	1 + 2 + 3	**4**	$\mathbf{2^3}$	
52	1 + 2 + 4	**4**	$\mathbf{2^3}$	
53	1 + 2 + 6	**4**	$\mathbf{2^3}$	
54	1 + 2 + 8	**4**	$\mathbf{2^3}$	
55	1 + 2 + 10	**4**	$\mathbf{2^3}$	
56	1 + 2 + 5	$\mathbf{2^2}$	$\mathbf{2^3}$	
57	1 + 2 + 7	$\mathbf{2^2}$	$\mathbf{2^3}$	
58	1 + 2 + 9	$\mathbf{2^2}$	$\mathbf{2^3}$	
59	1 + 2 + 12	$\mathbf{2^2}$	$\mathbf{2^3}$	
60	1 + 2 + 13	$\mathbf{2^2}$	$\mathbf{2^3}$	
61	1 + 2 + 14	$\mathbf{2^2}$	$\mathbf{2^3}$	
62	1 + 2 + 15	$\mathbf{2^2}$	$\mathbf{2^3}$	
63	1 + 2 + 16	$\mathbf{2^2}$	$\mathbf{2^3}$	
64	1 + 2 + 17	$\mathbf{2^2}$	$\mathbf{2^3}$	
65	1 + 2 + 11	$\mathbf{2^2}$	$\mathbf{2^3}$	
66	1 + 2	**2**	$\mathbf{2^4}$	$\mathbf{G' = Z = \Phi = \mathfrak{N}}$

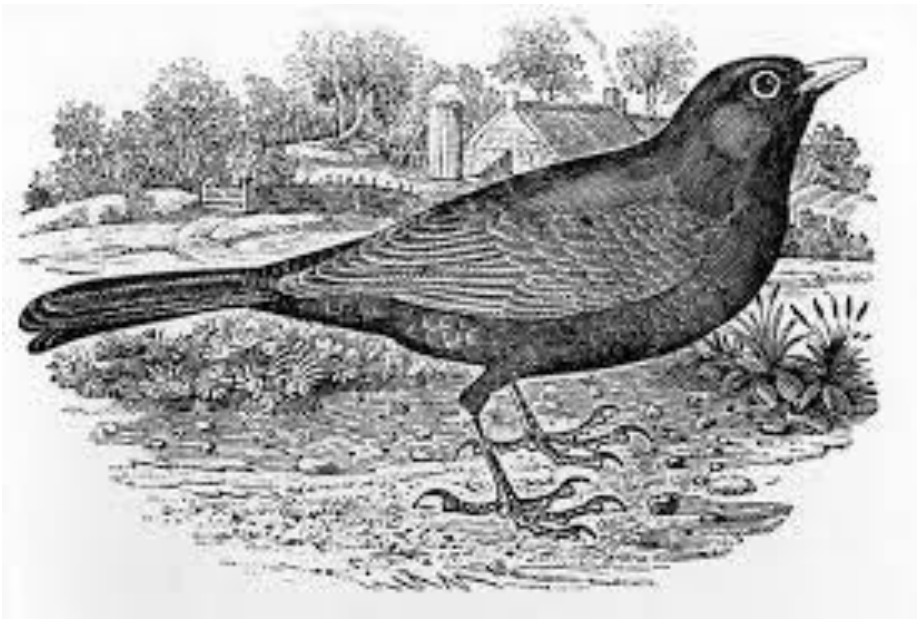
Maximal subgroups: [$\mathbf{D_8 \times 2}$] × 9, [**16.08**] × 6



32.43 = ⟨A^4, $B^2 = C^2 = A^2$, D^2, $CB = A^2BC$, $DA = A^{-1}D$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	A^2	A A^3	B A^2B	AB A^3B	C A^2C	AC A^3C	BC A^2BC	ABC A^3BC	D A^2CD	AD A^3D	BD A^2BD	ABD A^3BD	CD A^2CD	ACD A^3CD	BCD A^2BC D	ABCD A^3BC D

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
#	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	–1	–1	1	1	1	1	–1	–1	–1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	1	1	1	3
χ_5	1	1	1	–1	–1	1	1	–1	–1	1	1	–1	–1	1	1	–1	–1	4
χ_6	1	1	1	–1	–1	1	1	–1	–1	–1	–1	1	1	–1	–1	1	1	5
χ_7	1	1	1	–1	–1	–1	–1	1	1	1	1	–1	–1	–1	–1	1	1	6
χ_8	1	1	1	–1	–1	–1	–1	1	1	–1	–1	1	1	1	1	–1	–1	7
χ_9	1	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	8
χ_{10}	1	1	–1	1	–1	1	–1	1	–1	–1	1	–1	1	–1	1	–1	1	9
χ_{11}	1	1	–1	1	–1	–1	1	–1	1	1	–1	1	–1	–1	1	–1	1	10
χ_{12}	1	1	–1	1	–1	–1	1	–1	1	–1	1	–1	1	1	–1	1	–1	11
χ_{13}	1	1	–1	–1	1	1	–1	–1	1	1	–1	–1	1	1	–1	–1	1	12
χ_{14}	1	1	–1	–1	1	1	–1	–1	1	–1	1	1	–1	–1	1	1	–1	13
χ_{15}	1	1	–1	–1	1	–1	1	1	–1	1	–1	–1	1	–1	1	1	–1	14
χ_{16}	1	1	–1	–1	1	–1	1	1	–1	–1	1	1	–1	1	–1	–1	1	15
χ_{17}	4	–4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
^	1	2	4	4	2	4	2	4	2	2	2	4	4	4	4	4	4	



	Classes	H	G/H	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	**16.08**	**2**	1
2	1 + 2 + 3 + 4 + 5 + 10 + 11 + 12 + 13	**16.08**	**2**	2
3	1 + 2 + 3 + 4 + 5 + 14 + 15 + 16 + 17	$\mathbf{Q_8 \times 2}$	**2**	3
4	1 + 2 + 3 + 6 + 7 + 10 + 11 + 14 + 15	**16.08**	**2**	4
5	1 + 2 + 3 + 6 + 7 + 12 + 13 + 16 + 17	$\mathbf{Q_8 \times 2}$	**2**	5
6	1 + 2 + 3 + 8 + 9 + 10 + 11 + 16 + 17	**16.08**	**2**	6
7	1 + 2 + 3 + 8 + 9 + 12 + 13 + 14 + 15	$\mathbf{Q_8 \times 2}$	**2**	7
8	1 + 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16	$\mathbf{Q_8 \times 2}$	**2**	8
9	1 + 2 + 4 + 6 + 8 + 11 + 13 + 15 + 17	$\mathbf{Q_8 \times 2}$	**2**	9
10	1 + 2 + 4 + 7 + 9 + 10 + 12 + 15 + 17	**16.08**	**2**	10
11	1 + 2 + 4 + 7 + 9 + 11 + 13 + 14 + 16	**16.08**	**2**	11
12	1 + 2 + 5 + 6 + 9 + 10 + 13 + 14 + 17	**16.08**	**2**	12
13	1 + 2 + 5 + 6 + 9 + 11 + 12 + 15 + 16	**16.08**	**2**	13
14	1 + 2 + 5 + 7 + 8 + 10 + 13 + 15 + 16	**16.08**	**2**	14
15	1 + 2 + 5 + 7 + 8 + 11 + 12 + 14 + 17	**16.08**	**2**	15
16	1 + 2 + 3 + 4 + 5	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩2
17	1 + 2 + 3 + 6 + 7	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩4
18	1 + 2 + 3 + 8 + 9	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩6
19	1 + 2 + 4 + 6 + 8	$\mathbf{D_8}$	$\mathbf{2^2}$	1∩8
20	1 + 2 + 4 + 7 + 9	$\mathbf{D_8}$	$\mathbf{2^2}$	1∩10
21	1 + 2 + 5 + 6 + 9	$\mathbf{D_8}$	$\mathbf{2^2}$	1∩12
22	1 + 2 + 5 + 7 + 8	$\mathbf{D_8}$	$\mathbf{2^2}$	1∩14
23	1 + 2 + 3 + 10 + 11	$\mathbf{D_8}$	$\mathbf{2^2}$	2∩4
24	1 + 2 + 3 + 12 + 13	$\mathbf{D_8}$	$\mathbf{2^2}$	2∩5
25	1 + 2 + 4 + 10 + 12	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩8
26	1 + 2 + 4 + 11 + 13	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	2∩9
27	1 + 2 + 5 + 10 + 13	$\mathbf{D_8}$	$\mathbf{2^2}$	2∩12
28	1 + 2 + 5 + 11 + 12	$\mathbf{D_8}$	$\mathbf{2^2}$	2∩13
29	1 + 2 + 3 + 14 + 15	$\mathbf{D_8}$	$\mathbf{2^2}$	3∩4
30	1 + 2 + 3 + 16 + 17	$\mathbf{D_8}$	$\mathbf{2^2}$	3∩5
31	1 + 2 + 4 + 14 + 16	$\mathbf{D_8}$	$\mathbf{2^2}$	3∩8
32	1 + 2 + 4 + 15 + 17	$\mathbf{D_8}$	$\mathbf{2^2}$	3∩9
33	1 + 2 + 5 + 14 + 17	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩12
34	1 + 2 + 5 + 15 + 16	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	3∩13
35	1 + 2 + 6 + 10 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	4∩8
36	1 + 2 + 6 + 11 + 15	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	4∩9
37	1 + 2 + 7 + 10 + 15	$\mathbf{D_8}$	$\mathbf{2^2}$	4∩10
38	1 + 2 + 7 + 11 + 14	$\mathbf{D_8}$	$\mathbf{2^2}$	4∩11
39	1 + 2 + 6 + 12 + 16	$\mathbf{D_8}$	$\mathbf{2^2}$	5∩8
40	1 + 2 + 6 + 13 + 17	$\mathbf{D_8}$	$\mathbf{2^2}$	5∩9
41	1 + 2 + 7 + 12 + 17	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	5∩10
42	1 + 2 + 7 + 13 + 16	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	5∩11
43	1 + 2 + 8 + 10 + 16	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	6∩8
44	1 + 2 + 8 + 11 + 17	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	6∩9
45	1 + 2 + 9 + 10 + 17	$\mathbf{D_8}$	$\mathbf{2^2}$	6∩10

46	1 + 2 + 9 + 11 + 16	$\mathbf{D_8}$	$\mathbf{2^2}$	6∩11
47	1 + 2 + 8 + 12 + 14	$\mathbf{D_8}$	$\mathbf{2^2}$	7∩8
48	1 + 2 + 8 + 13 + 15	$\mathbf{D_8}$	$\mathbf{2^2}$	7∩9
49	1 + 2 + 9 + 12 + 15	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	7∩10
50	1 + 2 + 9 + 13 + 14	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	7∩11
51	1 + 2 + 3	**4**	$\mathbf{2^3}$	16∩17
52	1 + 2 + 4	**4**	$\mathbf{2^3}$	19∩20
53	1 + 2 + 5	$\mathbf{2^2}$	$\mathbf{2^3}$	33∩34
54	1 + 2 + 6	**4**	$\mathbf{2^3}$	35∩36
55	1 + 2 + 7	$\mathbf{2^2}$	$\mathbf{2^3}$	37∩38
56	1 + 2 + 8	**4**	$\mathbf{2^3}$	43∩44
57	1 + 2 + 9	$\mathbf{2^2}$	$\mathbf{2^3}$	45∩46
58	1 + 2 + 10	$\mathbf{2^2}$	$\mathbf{2^3}$	43∩45
59	1 + 2 + 11	$\mathbf{2^2}$	$\mathbf{2^3}$	44∩46
60	1 + 2 + 12	**4**	$\mathbf{2^3}$	47∩49
61	1 + 2 + 13	**4**	$\mathbf{2^3}$	48∩50
62	1 + 2 + 14	**4**	$\mathbf{2^3}$	47∩50
63	1 + 2 + 15	**4**	$\mathbf{2^3}$	48∩49
64	1 + 2 + 16	**4**	$\mathbf{2^3}$	42∩43
65	1 + 2 + 17	**4**	$\mathbf{2^3}$	44∩45
66	1 + 2	**2**	$\mathbf{2^4}$	

$\mathbf{G' = Z = \Phi = \mathfrak{R}}$

Maximal subgroups: [$\mathbf{Q_8 \times 2}$] × 5, [**16.08**] × 10

32.44 = ⟨A^8, B^2, C^2, $BA = A^5B$, $CA = A^{-1}C$⟩

1	2	3	4	5	6	7	8	9	10	11
1	A^4	A^2 A^6	B A^4B	A^2B A^6B	A^{2n+1}	$A^{2n+1}B$	$A^{2n}C$	$A^{2n}BC$	$A^{2n+1}C$	$A^{2n+1}BC$

C	1	2	3	4	5	6	7	8	9	10	11	
#	1	1	2	2	2	4	4	4	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	–1	–1	1	–1	1	–1	1	–1	4
χ_6	1	1	1	–1	–1	1	–1	–1	1	–1	1	5
χ_7	1	1	1	–1	–1	–1	1	1	–1	–1	1	6
χ_8	1	1	1	–1	–1	–1	1	–1	1	1	–1	7
χ_9	2	2	–2	2	–2	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	2	0	0	0	0	0	0	9
χ_{11}	4	–4	0	0	0	0	0	0	0	0	0	0
^	1	2	4	2	4	8	8	2	2	2	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	$\mathbf{M}_{8,2}^{(5)}$	2	1	
2	1 + 2 + 3 + 4 + 5 + 8 + 9	$\mathbf{D}_8 \times 2$	2	2	
3	1 + 2 + 3 + 4 + 5 + 10 + 11	16.08	2	3	
4	1 + 2 + 3 + 6 + 8 + 10	$\mathbf{D}_{16}$	2	4	
5	1 + 2 + 3 + 6 + 9 + 11	$\mathbf{M}_{8,2}^{(3)}$	2	5	
6	1 + 2 + 3 + 7 + 8 + 11	$\mathbf{M}_{8,2}^{(3)}$	2	6	
7	1 + 2 + 3 + 7 + 9 + 10	$\mathbf{D}_{16}$	2	7	
8	1 + 2 + 4	2^2	$\mathbf{D}_8$	8	
9	1 + 2 + 5	4	$\mathbf{D}_8$	9	
10	1 + 2 + 3 + 4 + 5	4×2	2^2	1∩2	$\mathbf{Z}_2$
11	1 + 2 + 3 + 6	8	2^2	1∩4	
12	1 + 2 + 3 + 7	8	2^2	1∩6	
13	1 + 2 + 3 + 8	$\mathbf{D}_8$	2^2	2∩4	
14	1 + 2 + 3 + 9	$\mathbf{D}_8$	2^2	2∩5	
15	1 + 2 + 3 + 10	$\mathbf{D}_8$	2^2	3∩4	
16	1 + 2 + 3 + 11	$\mathbf{Q}_8$	2^2	3∩5	
17	1 + 2	2	$\mathbf{D}_8 \times 2$	4∩8	$\mathbf{Z} = \mathfrak{Z}$
18	1 + 2 + 3	4	2^3	11∩12	$\mathbf{G'} = \mathbf{\Phi}$

Maximal subgroups: [$\mathbf{D}_8 \times 2$], [**16.08**], [$\mathbf{M}_{8,2}^{(5)}$], [$\mathbf{D}_{16}$], [$\mathbf{M}_{8,2}^{(3)}$] × 3

32.45 = $\langle A^8, B^2, C^2 = A^4, BA = A^5B, CA = A^{-1}C \rangle$

1	2	3	4	5	6	7	8	9	10	11
1	A^4	A^2 A^6	B A^4B	A^2B A^6B	A^{2n+1}	$A^{2n+1}B$	$A^{2n}C$	$A^{2n}BC$	$A^{2n+1}C$	$A^{2n+1}BC$

C	1	2	3	4	5	6	7	8	9	10	11	
#	1	1	2	2	2	4	4	4	4	4	4	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	–1	–1	1	–1	1	–1	1	–1	4
χ_6	1	1	1	–1	–1	1	–1	–1	1	–1	1	5
χ_7	1	1	1	–1	–1	–1	1	1	–1	–1	1	6
χ_8	1	1	1	–1	–1	–1	1	–1	1	1	–1	7
χ_9	2	2	–2	2	–2	0	0	0	0	0	0	8
χ_{10}	2	2	–2	–2	2	0	0	0	0	0	0	9
χ_{11}	4	–4	0	0	0	0	0	0	0	0	0	0
∧	1	2	4	2	4	8	8	4	4	4	2	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	$\mathbf{M_{8,2}^{(5)}}$	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 8 + 9	$\mathbf{Q_8 \times 2}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 10 + 11	**16.08**	**2**	3	
4	1 + 2 + 3 + 6 + 8 + 10	$\mathbf{Q_{16}}$	**2**	4	
5	1 + 2 + 3 + 6 + 9 + 11	$\mathbf{M_{8,2}^{(3)}}$	**2**	5	
6	1 + 2 + 3 + 7 + 8 + 11	$\mathbf{M_{8,2}^{(3)}}$	**2**	6	
7	1 + 2 + 3 + 7 + 9 + 10	$\mathbf{Q_{16}}$	**2**	7	
8	1 + 2 + 4	$\mathbf{2^2}$	$\mathbf{D_8}$	8	
9	1 + 2 + 5	**4**	$\mathbf{D_8}$	9	$\mathcal{N}$
10	1 + 2 + 3 + 4 + 5	$\mathbf{4 \times 2}$	$\mathbf{2^2}$	1∩2	$\mathbf{Z_2}$
11	1 + 2 + 3 + 6	**8**	$\mathbf{2^2}$	1∩4	
12	1 + 2 + 3 + 7	**8**	$\mathbf{2^2}$	1∩6	
13	1 + 2 + 3 + 8	$\mathbf{Q_8}$	$\mathbf{2^2}$	2∩4	
14	1 + 2 + 3 + 9	$\mathbf{Q_8}$	$\mathbf{2^2}$	2∩5	
15	1 + 2 + 3 + 10	$\mathbf{Q_8}$	$\mathbf{2^2}$	3∩4	
16	1 + 2 + 3 + 11	$\mathbf{D_8}$	$\mathbf{2^2}$	3∩5	
17	1 + 2	**2**	$\mathbf{D_8 \times 2}$	4∩8	**Z**
18	1 + 2 + 3	**4**	$\mathbf{2^3}$	10∩11	**G′ = Φ**

Maximal subgroups: [$\mathbf{Q_8 \times 2}$], [**16.08**], [$\mathbf{M_{8,2}^{(5)}}$], [$\mathbf{M_{8,2}^{(3)}}$] × 2, [$\mathbf{Q_{16}}$] × 2

32.46 = ⟨A^2, B^2, C^2, D^4, DB = ABD, DC = ABCD⟩

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	A	B AB	D^2 AD^2	BD^2 ABD^2	A^mB^nC	$A^mB^nCD^2$	A^mB^nD	$A^mB^nD^3$	A^mB^nCD	$A^mB^nCD^3$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	
#	**1**	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**4**	**4**	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	–1	–1	1	–1	i	–i	i	–i	4
χ_6	1	1	1	–1	–1	1	–1	–i	i	–i	i	4
χ_7	1	1	1	–1	–1	–1	1	i	–i	–i	i	5
χ_8	1	1	1	–1	–1	–1	1	–i	i	i	–i	5
χ_9	2	2	–2	2	–2	0	0	0	0	0	0	6
χ_{10}	2	2	–2	–2	2	0	0	0	0	0	0	7
χ_{11}	4	–4	0	0	0	0	0	0	0	0	0	0
^	**1**	**2**	**2**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**4**	

	Classes	**H**	**G/H**	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	**$D_8 \times 2$**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 8 + 9	**16.09**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 10 + 11	**16.09**	**2**	3	
4	1 + 2 + 3 + 6	**2^3**	**4**	4	
5	1 + 2 + 3 + 6 + 7	**4×2**	**4**	5	
6	1 + 2 + 4	**2^2**	**D_8**	6	
7	1 + 2 + 5	**2^2**	**D_8**	7	
8	1 + 2 + 3 + 4 + 5	**2^3**	**2^2**	1∩2	**$Z_2 = \Phi$**
9	1 + 2 + 3	**2^2**	**4×2**	2∩4	**G′**
10	1 + 2	**2**	**16.09**	4∩6	**Z = 𝔄**

Maximal subgroups: [**$D_8 \times 2$**], [**16.09**] × 2

32.47 = ⟨A^8, B^2, C^2, $BA = A^{-1}B$, $CA = ABC$⟩

1	2	3	4	5	6	7	8	9	10	11
1	A^4	B A^4B	A^2 A^6	A^2B A^6B	C BC A^4C A^4BC	A^2C A^2BC A^6C A^6BC	A AB A^5 A^5B	A^3 A^3B A^7 A^7B	AC ABC A^5C A^5BC	A^3C A^3BC A^7C A^7BC

C	1	2	3	4	5	6	7	8	9	10	11	
#	1	1	2	2	2	4	4	4	4	4	4	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	–1	–1	1	–1	i	–i	i	–i	4
χ_6	1	1	1	–1	–1	1	–1	–i	i	–i	i	4
χ_7	1	1	1	–1	–1	–1	1	i	–i	–i	i	5
χ_8	1	1	1	–1	–1	–1	1	–i	i	i	–i	5
χ_9	2	2	–2	2	–2	0	0	0	0	0	0	6
χ_{10}	2	2	–2	–2	2	0	0	0	0	0	0	7
χ_{11}	4	–4	0	0	0	0	0	0	0	0	0	0
^	1	2	2	4	4	2	2	8	8	8	8	

	Classes	Normal subgroups H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	**$D_8 \times 2$**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 8 + 9	**$M_{8,2}^{(5)}$**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 10 + 11	**$M_{8,2}^{(5)}$**	**2**	3	
4	1 + 2 + 3 + 6	**2^3**	**4**	4	
5	1 + 2 + 3 + 7	**2^3**	**4**	5	
6	1 + 2 + 4	**4**	**D_8**	6	
7	1 + 2 + 5	**4**	**D_8**	7	
8	1 + 2 + 3 + 4 + 5	**4×2**	**2^2**	1∩2	**$Z_2 = \Phi$**
9	1 + 2 + 3	**2^2**	**4×2**	2∩4	**$G' = \mathfrak{A}$**
10	1 + 2	**2**	**16.09**	4∩6	**Z**

Maximal subgroups: [**$D_8 \times 2$**], [**$M_{8,2}^{(5)}$**] × 2

32.48 = ⟨A^8, B^2, C^2, $BA = A^5B$, $CA = ABC$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	A^4	B A^4B	A^2 A^6	A^2B A^6B	C BC A^4C A^4BC	A^2C A^2BC A^6C A^6BC	A AB A^5 A^5B	A^3 A^3B A^7 A^7B	AC ABC A^5C A^5BC	A^3C A^3BC A^7C A^7BC

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	
#	**1**	**1**	**2**	**2**	**2**	**4**	**4**	**4**	**4**	**4**	**4**	$\mathfrak{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	–1	–1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	1	1	–1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	1	1	3
χ_5	1	1	1	–1	–1	1	–1	i	–i	i	–i	4
χ_6	1	1	1	–1	–1	1	–1	–i	i	–i	i	4
χ_7	1	1	1	–1	–1	–1	1	i	–i	–i	i	5
χ_8	1	1	1	–1	–1	–1	1	–i	i	i	–i	5
χ_9	2	2	–2	2	–2	0	0	0	0	0	0	6
χ_{10}	2	2	–2	–2	2	0	0	0	0	0	0	7
χ_{11}	4	–4	0	0	0	0	0	0	0	0	0	0
^	**1**	**2**	**2**	**4**	**4**	**4**	**4**	**8**	**8**	**8**	**8**	

	Classes	**H**	**G/H**	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	$Q_8 \times 2$	2	1	
2	1 + 2 + 3 + 4 + 5 + 8 + 9	$M_{8,2}^{(5)}$	2	2	
3	1 + 2 + 3 + 4 + 5 + 10 + 11	$M_{8,2}^{(5)}$	2	3	
4	1 + 2 + 3 + 6	4×2	4	4	
5	1 + 2 + 3 + 7	4×2	4	5	
6	1 + 2 + 4	4	D_8	6	
7	1 + 2 + 5	4	D_8	7	
8	1 + 2 + 3 + 4 + 5	4×2	2^2	1∩2	$\mathbf{Z_2} = \mathbf{\Phi} = \mathfrak{R}$
9	1 + 2 + 3	2^2	4×2	2∩4	**G′**
10	1 + 2	2	16.09	4∩6	**Z**

Maximal subgroups: [$\mathbf{Q}_8 \times 2$], [$\mathbf{M}_{8,2}^{(5)}$] × 2

32.49 = D_{32} = $\langle A^{16}, B^2, BA = A^{-1}B \rangle$

1	2	3	4	5	6	7	8	9	10	11
1	A^8	A^4 A^{12}	A^2 A^{14}	A^6 A^{10}	A A^{15}	A^7 A^9	A^5 A^{11}	A^2 A^{13}	$A^{2n}B$	$A^{2n+1}B$

C	1	2	3	4	5	6	7	8	9	10	11	
#	1	1	2	2	2	2	2	2	2	8	8	$\mathcal{K}$
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	–1	–1	1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	–1	1	3
χ_5	2	2	2	–2	–2	0	0	0	0	0	0	4
χ_6	2	2	–2	0	0	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	0	0	5
χ_7	2	2	–2	0	0	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	0	0	5
χ_8	2	–2	0	$\sqrt{2}$	$-\sqrt{2}$	2i	–2i	–2i	2i	0	0	0
χ_9	2	–2	0	$\sqrt{2}$	$-\sqrt{2}$	–2i	2i	2i	–2i	0	0	0
χ_{10}	2	–2	0	$-\sqrt{2}$	$\sqrt{2}$	2i	–2i	2i	–2i	0	0	0
χ_{11}	2	–2	0	$-\sqrt{2}$	$\sqrt{2}$	–2i	2i	–2i	2i	0	0	0
^	1	2	4	8	8	16	16	16	16	2	2	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	**16**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 10	$\mathbf{D_{16}}$	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 11	$\mathbf{D_{16}}$	**2**	3	
4	1 + 2 + 3	**4**	$\mathbf{D_8}$	4	$\mathbf{Z_2}$
5	1 + 2	**2**	$\mathbf{D_{16}}$	5	$\mathbf{Z} = \mathfrak{A}$
6	1 + 2 + 3 + 4 + 5	**8**	$\mathbf{2^2}$	1∩2	$\mathbf{Z_3 = G' = \Phi}$

Maximal subgroups: [**16**], [$\mathbf{D_{16}}$] × 2

32.50 = $M_{16,2}^{(7)}$ = ⟨A^{16}, B^2, $BA = A^7B$⟩

1	2	3	4	5	6	7	8	9	10	11
1	A^8	A^4 A^{12}	A^6 A^{10}	A^2 A^{14}	A A^7	A^9 A^{15}	A^3 A^5	A^{11} A^{13}	$A^{2n}B$	$A^{2n+1}B$

C	1	2	3	4	5	6	7	8	9	10	11	
#	1	1	2	2	2	2	2	2	2	8	8	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	–1	–1	1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	–1	1	3
χ_5	2	2	2	–2	–2	0	0	0	0	0	0	4
χ_6	2	2	–2	0	0	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	0	0	5
χ_7	2	2	–2	0	0	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	0	0	5
χ_8	2	–2	0	$\sqrt{2}$	$-\sqrt{2}$	2ci	–2ci	–2si	2si	0	0	0
χ_9	2	–2	0	$\sqrt{2}$	$-\sqrt{2}$	–2ci	2ci	2si	–2si	0	0	0
χ_{10}	2	–2	0	$-\sqrt{2}$	$\sqrt{2}$	2si	–2si	2ci	–2ci	0	0	0
χ_{11}	2	–2	0	$-\sqrt{2}$	$\sqrt{2}$	–2si	2si	–2ci	2ci	0	0	0
order	1	2	4	8	8	16	16	16	16	2	2	

c = cos(2π/16) = ½$\sqrt{2 + \sqrt{2}}$, s = sin(2π/16) = ½$\sqrt{2 - \sqrt{2}}$

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	16	2	1	
2	1 + 2 + 3 + 4 + 5 + 10	$\mathbf{D}_{16}$	2	2	
3	1 + 2 + 3 + 4 + 5 + 11	$\mathbf{D}_{16}$	2	3	
4	1 + 2 + 3	4	$\mathbf{D}_8$	4	$\mathbf{Z_2}$
5	1 + 2	2	$\mathbf{D}_{16}$	5	$\mathbf{Z} = \mathfrak{A}$
6	1 + 2 + 3 + 4 + 5	8	2^2	1∩2	$\mathbf{Z_3 = G' = \Phi}$

Maximal subgroups: [**16**], [$\mathbf{D}_{16}$], [$\mathbf{Q}_{16}$]

32.51 = Q_{32} = ⟨A^{16}, $B^2 = A^8$, $BA = A^{-1}B$⟩

1	2	3	4	5	6	7	8	9	10	11
1	A^8	A^4 A^{12}	A^2 A^{14}	A^6 A^{10}	A A^{15}	A^7 A^9	A^5 A^{11}	A^3 A^{13}	$A^{2n}B$	$A^{2n+1}B$

C	1	2	3	4	5	6	7	8	9	10	11	
#	1	1	2	2	2	2	2	2	2	8	8	ℵ
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	1	–1	–1	1
χ_3	1	1	1	1	1	–1	–1	–1	–1	1	–1	2
χ_4	1	1	1	1	1	–1	–1	–1	–1	–1	1	3
χ_5	2	2	2	–2	–2	0	0	0	0	0	0	4
χ_6	2	2	–2	0	0	√2	√2	–√2	–√2	0	0	5
χ_7	2	2	–2	0	0	–√2	–√2	√2	√2	0	0	5
χ_8	2	–2	0	√2	–√2	2i	–2i	–2i	2i	0	0	0
χ_9	2	–2	0	√2	–√2	–2i	2i	2i	–2i	0	0	0
χ_{10}	2	–2	0	–√2	√2	2i	–2i	2i	–2i	0	0	0
χ_{11}	2	–2	0	–√2	√2	–2i	2i	–2i	2i	0	0	0
^	1	2	4	8	8	16	16	16	16	4	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	16	2	1	
2	1 + 2 + 3 + 4 + 5 + 10	Q_{16}	2	2	
3	1 + 2 + 3 + 4 + 5 + 11	Q_{16}	2	3	
4	1 + 2 + 3	4	Q_8	4	
5	1 + 2	2	Q_{16}	5	**Z**
6	1 + 2 + 3 + 4 + 5	8	2^2	1∩2	$Z_2 = G' = \Phi = \mathfrak{A}$

Maximal subgroups: [**16**], [Q_{16}] × 2